

## Credibility and the Common-Sense Actuary

**T**HE FOLLOWING DIALOGUE IS HYPOTHETICAL. The antagonists, both fellows of the Society of Actuaries, are an actuarial consultant and the employee of a group health insurer. The subject is a group long-term-care case:

**Consultant:** *How would you say you'd experience-rate this group?*

**Insurer:** *I can't say that I would experience-rate this group.*

**Consultant:** *You can't because you don't want to or don't know how to?*

**Insurer:** *Because I wouldn't know how to.*

**Consultant:** *Well, what credibility do you assign this group's experience?*

**Insurer:** *None. We don't experience-rate any group. We apply our standard pricing assumptions to every group.*

**Consultant** (incredulous—maybe even self-righteous): *Not even a 50,000-life group with 100 percent participation?*

**Insurer:** *That's correct. We don't believe any group has credible experience. We only ascribe credibility to our entire block of business.*

Let's depart from this brief conversation before the exchange gets testy. Our hypothetical consultant's incredulity is understandable. Who in the world with an ounce of common sense would assign no credibility to a 50,000-life case? Moreover, wouldn't common sense dictate that the credibility be 100 percent?

In fact, depending on the type of coverage, the insurer's position could be quite defensible. Sometimes mathematical reasoning points to a conclusion that runs counter to common sense.

### Counterexample vs. Common Sense

The actuary and the layman don't use the term "credibility" in the same sense. To the layman, credibility is a fuzzy term having something to do with whether data can be trusted. The actuary uses the term to denote a factor  $Z$ , which is the weight assigned to the group's actual experience. The rate the group will be charged is a weighted

average of two rates, one based on the group's experience, the other drawn from the insurer's rate manual:

$$\text{Group Rate} = Z \cdot (\text{Experience Rate}) + (1-Z) \cdot (\text{Manual Rate}).$$

When  $Z=1$ , the data is 100 percent credible. The lay notion and the actuarial concept are different, but they do overlap. Common sense is partly right. When the number of exposed lives goes up, all other things being equal, the credibility goes up, in both the lay sense and the actuarial sense of the term.

There is something that common sense might miss. As the probability of the claim goes down, the credibility also goes down. I'll illustrate with an extreme example. If the experience on 50,000 exposed lives is fully credible, then the experience on 2 million exposed lives also must be fully credible. Suppose we have a group of 2 million lives with an expected annual rate of claim of one in 1 million. On such a group, we expect to incur two claims in a single year. If we actually incur only one claim, should we slash the group's rate by 50 percent?



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Or if we incur three claims, should we raise the group's rate by 50 percent? What does common sense tell us now?

In this example, the probability that the experience rate will be only 50 percent of the expected rate is quite high. Since we are conducting 2 million independent repetitions of an experiment with only two possible outcomes (claim/no claim), each repetition having a one in 1 million chance of "success," the total number of claims will follow a binomial distribution.

It's hard to apply the formula for the binomial distribution to this case (watch what Excel does when you try to calculate 2 million factorial), but we have an acceptable approximation. As the number of repetitions gets very large and the probability of success on any repetition gets very small, the binomial distribution converges to the Poisson distribution

$$\Pr(X=k) = e^{-\lambda} \lambda^k/k!,$$

where  $\lambda$  is the number of expected claims. In this case  $\lambda = 2$ . The probability of only one claim is then approximately

$$e^{-2}2^1/1! \approx 27\%.$$

And with a little luck, the group can obtain insurance coverage free of charge: The chance of zero claims in a year is a little over 13 percent. Common sense to the contrary, this very large group's experience has no credibility at all.

**Variance vs. Credibility**

Why does credibility go down as the probability of a claim goes down? To investigate, let's define some notation and make some assumptions.

- ▶  $n$  = number of exposed lives.
- ▶  $p$  = the expected claim frequency; that is, the probability an exposed life will incur a claim within some specified time period.
- ▶  $M$  = a random variable, the number of claims. Note that  $E(M) = np$  and  $\sigma^2(M) = np(1-p)$ .
- ▶  $X_i$  = a random variable, the size of the  $i$ -th claim.

Then the total claim cost incurred over the time period for which the claim frequency is defined would be

$$X_1 + X_2 + \dots + X_M,$$

where the final subscript is a random variable.

We will make three standard assumptions. First, claim severity is independent of the number of claims; that is, each random variable  $X_i$  is independent of  $M$ . Second, if  $i \neq j$ , then  $X_i$  is independent of  $X_j$ . Third, the  $X_i$  are identically distributed, so that, for all  $i$ ,  $E(X_i) = E(X_1)$  and  $\sigma^2(X_i) = \sigma^2(X_1)$ .

From these assumptions, it follows that the mean claim cost will be:

$$E(X_1 + X_2 + \dots + X_M) = E(M) \cdot E(X_1) = npE(X_1).$$



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We know from risk theory that the variance of the total claim cost will be:

$$\sigma^2(X_1 + X_2 + \dots X_M) = E(M) \cdot \sigma^2(X_1) + \sigma^2(M) \cdot E(X_1)^2 = np\sigma^2(X_1) + np(1-p)E(X_1)^2.$$

In order to consider experience data to be credible, we require that, with high probability

$$|X_1 + X_2 + \dots X_M - E(X_1 + X_2 + \dots X_M)| < rE(X_1 + X_2 + \dots X_M),$$

where r is some very small fraction. In words, we consider experience data credible if, with high probability, the claim experience observed will be very close to the expected claim cost. Notice that “very close” is defined as a fraction of the mean. Dividing both sides of the inequality by  $E(X_1 + X_2 + \dots X_M) = npE(X_1)$ , we get

$$\left| \frac{X_1 + X_2 + \dots X_M}{npE(X_1)} - 1 \right| < r$$

In other words, data is credible if, with high probability, the actual-to-expected claim cost is close to one. Notice that one is the mean of the actual-to-expected claim cost. As we know from the Chebychev Inequality ( $\Pr[|Y - E(Y)| \geq \epsilon] \leq \sigma^2(Y)/\epsilon^2$ ), if the variance of a random variable is sufficiently small, that random variable has a high likelihood of realizing a value close to its mean. High credibility should therefore go hand in hand with low

variance for the actual-to-expected claim cost.

So, let’s investigate what happens to the variance of the actual-to-expected claim cost if the claim frequency p gets very small:

$$\sigma^2\left(\frac{X_1 + X_2 + \dots X_M}{npE(X_1)}\right) = \frac{np\sigma^2(X_1) + np(1-p)E(X_1)^2}{(npE(X_1))^2} = \frac{\sigma^2(X_1)}{npE(X_1)^2} + \frac{1-p}{n}$$

There are three facts you should note about both terms on the right-hand side.

- ▶ If  $p \rightarrow 0$ , both terms approach infinity. That explains why a small expected frequency implies very low credibility: The variance of the actual-to-expected claim cost grows, and we lose the assurance that the actual claim cost will, with high probability, fall close to the mean.
- ▶ If  $n \rightarrow \infty$ , then both terms approach zero. That fact supports our common-sense notion that, as the number of exposed lives increases, the credibility of the data should also increase. In this latter case, Chebychev’s inequality will imply that the experience claim cost will be close to the mean with probability approaching 100 percent.
- ▶ If the ratio  $\sigma^2(X_1)/E(X_1)^2$  increases, then the first term in the expression for the variance of the actual-to-expected ratio increases. In other words, volatility in the claim severity will also reduce the credibility of experience data. Notice that  $np$ , the number of expected claims, occurs in the denominator of the first term. That

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is, an increase in the expected claims can offset an increase in the volatility of the claim severity to restore data credibility.

**Credibility vs. Common Sense**

We have three factors—expected claim frequency, life exposure, and the ratio of the variance to the mean of claim severity—that affect data credibility. It would be desirable to have a single measure that reflects all factors. The measure utilized in classical limited fluctuation credibility is the number of expected claims. As life exposure goes up, so does the credibility, and so does the number of expected claims. As the expected claim frequency goes down, so does the credibility, and so does the number of expected claims. And, as observed above, if the number of expected claims is high enough, it offsets the volatility in the claim severity.

Classical limited fluctuation credibility uses the same criteria I used in the prior section. It identifies a group's experience as fully credible when, with a high degree of probability, the experience claim cost will fall inside a narrow confidence interval around the mean claim cost. When a group's experience isn't fully credible, it's assigned partial credibility, equated to the square root of the ratio of the number of expected claims to the number required for full credibility.

As in the prior section, the size of the confidence interval is measured as a fraction of the mean claim cost. That is, if  $c$  is the expected claim cost and  $C$  the experience claim cost, then a group's experience is fully credible when

$$\Pr(|C-c| < rc) \geq s.$$

In the above inequality,  $r$  is some small fraction and  $s$  is a high probability. For example, if  $r=.05$  and  $s=90$  percent, then the expected number of claims must be at least 1,082 to ensure full credibility, assuming there is no variation in the size of claim. When there is variation in the claim severity, the number of expected claims required for full credibility is increased by the factor

$$\left(1 + \frac{\sigma^2(X)}{E^2(X)}\right),$$

where  $\sigma^2(X)$  is the variance in the size of claim and  $E(X)$  is the mean size of the claim. Again, all claims are assumed identically distributed, thus sharing a common mean and standard deviation. Any reader interested in rigorous derivation of these facts should consult the (excellent) text by Herzog.

So, let's apply credibility theory to a case with 50,000 lives. Initially, we'll take 1,082 expected claims as the threshold for 100 percent credibility. If the average age of the group is in the 40s, then, for long-term care, the expected rate of claim could be extremely low, perhaps only a few claims for every 10,000 exposed lives. For the sake of argument, let's assume a rate of 10 claims for every 10,000 exposed lives. In one year, we expect to incur 50 claims, and the credibility of a single year of data is then

$$Z = \sqrt{50/1082} = 21\%.$$

In two years, we expect 100 claims, so the credibility of two

years of data is a mere

$$Z = \sqrt{100/1082} = 30\%.$$

In fact, it takes six years of data just to get the credibility higher than 50 percent, assuming we can ignore the variance in the size of a claim. But we really shouldn't ignore that extra element of uncertainty. For long-term care, the variation in claim severity is substantial. Some claimants die shortly after completing the waiting period; others languish in a nursing home for years. Some draw down their benefit pool at the maximum possible rate; others utilize services intermittently.

Claim-severity assumptions dictate how large the correction should be. In the appendix, I create a scenario that's simplistic but not entirely outlandish. My purpose is to demonstrate that variation in size of claim is significant enough to affect the criteria for full credibility. If accepted, the scenario implies that the standard of 100 percent credibility should be

$$1082 \cdot (1 + 0.7^2) = 1,612.$$

In that case, it will take eight years for our 50,000-life case to reach 50 percent credibility. And there are other complications.

### One Group vs. Many Risk Classes

One complication is that not all members of the same insured

group are members of the same risk class. To derive the variance of the total claim cost incurred,

$$X_1 + X_2 + \dots + X_M,$$

we assumed the  $X_i$  were identically distributed. In other words, at least in regard to claim severity, we assumed that all lives fell into the same risk class. For group long-term care, uniformity of risk is a serious problem because the premium isn't calculated from a single aggregate rate applied to the entire group but is the sum of premiums calculated for each individual life in the group. Suppose our 50,000-life case divides neatly into two subgroups. One subgroup contains 25,000 members who have yet to see their 30th birthday. The other subgroup contains 25,000 members who are only a few years from retirement. Should we calculate a single credibility factor for the entire group or separate credibility factors for each subgroup? My common sense suggests the latter course of action.

### Credibility vs. Relevance

The final complication is the most important of all. Group long-term care is unique for the lag (about three decades) between the average age at which the product is purchased and the average age when claims will be incurred. To evaluate long-term-care pricing assumptions, the actuary needs credible claim experience on lives

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that are well past the age of 70; claim experience on individuals in their 40s might be credible but not very relevant to the question of rate adequacy. That's why, in my opinion, no group long-term-care case should be experience rated. It's just—um—common sense.

Experience rating works well when the group is charged a single aggregate premium that is frequently reviewed and revised. It doesn't work so well to prefund claims decades into the future when each member is charged an individual premium that's intended to remain level for a lifetime. I recognize that other actuaries may have a reasonable disagreement with my position.

### Theory vs. Practice

I've noted four attributes of group long-term care that severely limit the actuary's ability to experience-rate a case: the low claim incidence rate, the high variability in the size of a claim, the need to divide a group into homogeneous risk classes, and the long delay from issue to claim incurral. Not all group lines of insurance share these attributes (excepting the need for homogeneous risk classes). Nevertheless, my discussion may have some relevance to group products in general.

It's common practice for group life and health insurers to use a less demanding credibility standard than theory would justify. Between theory and practice lies the competitive pressure of the marketplace. Actuaries cannot ignore market forces, but neither

should they ignore the effect an irrational pricing strategy may have on the public. Do we penalize a group by assigning too much credibility to what is simply bad luck? Do we expose the marketplace to too much rate instability when "credible data" failed to foretell future adverse experience? Are we really serving our employers well when we ape the competition's pricing strategy? The actuary who says he can serve two masters, his employer and the public, may serve neither master very well.

Actuarial consultants also make choices. If it's in the client's best interest, can the consultant argue for a level of credibility that lacks solid theoretical justification? Is the actuary always an objective scientist, or does he ever advocate for his client, much as a lawyer zealously presses a client's case, right or wrong? (There are many lessons the actuarial consultant may, at some future date, learn from the lawyer. Prancing before the client to justify a high hourly rate and to increase billable hours is one that comes to mind.)

My own hope is that the public never equates the actuary with the lawyer. Should that ever come to pass, the profession will suffer a serious loss of credibility, in the layman's sense of that term. ●

### Reference

Herzog, Thomas N., *Introduction to Credibility Theory*, ACTEX Publications, 1999

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## Appendix

Possible value for  $\sigma(X)/E(X)$ , coefficient of variation in claim severity

The following assumptions are simplistic. Their purpose is only to demonstrate that the coefficient of variation for long-term care is large enough to produce a significant reduction in data credibility. Assume that:

- 1) Every group member has a single plan in which the benefit pool, if drawn down at the maximum rate, would last 60 months.
- 2) The force of decrement from claim is constant; the claim termination rate is 2 percent per month. The constant force of decrement (expressed for monthly termination, not annual termination) is then  $\mu_t = -\ln(1-.02)$ .
- 3) Half of all claimants begin in facility care, drawing down their benefit pool at the maximum rate (here taken to be \$1.00 a month).
- 4) The remaining half of claimants begin by receiving services at home, drawing down their benefit pool at \$0.50 a month. (The lower payout rate means they can remain on claim for 120 months before exhausting their benefit pool.)
- 5) There's no migration between facility care and home care: Claimants who start in facility care never go home; claimants at home never enter a facility.

Assumptions 2 and 5 aren't realistic. Migration from home care to facility care is common and the reverse not unheard of. The force of mortality will not remain constant over a period of years, nor will it be the same for claimants in facilities and claimants at home. I stress again that I've chosen these assumptions merely to illustrate the possible magnitude of the coefficient of variation.

Define random variable X to be the claim severity, measured in dollars paid. Define random variable I to be 0 if the claim begins in facility care and 1 if the claim begins at home. Let T be the time on claim, expressed in months. If I=0, then X=T; if I=1, then X=0.5 • T. Per assumptions 3 and 4,  $\Pr[I=0]=\Pr[I=1]=1/2$ .

$$E(X | I = 0) = \int_{t=0}^{t=60} (1 - .02)^t dt = 34.8 \text{ dollars,}$$

$$E(X | I = 1) = \int_{t=0}^{t=120} 0.5 \cdot (1 - .02)^t dt = 22.6 \text{ dollars,}$$

$$E(X) = E_I E(X | I) = .5 \cdot 34.8 + .5 \cdot 22.6 = 28.7 \text{ dollars,}$$

$$E(X^2 | I = 0) = \int_{t=0}^{t=60} t^2 (1 - .02)^t \mu_t dt + 60^2 (1 - .02)^{60} = 1674.7 \text{ dollars}^2,$$

$$E(X^2 | I = 1) = \int_{t=0}^{t=120} (0.5 \cdot t)^2 (1 - .02)^t \mu_t dt + (0.5 \cdot 120)^2 (1 - .02)^{120} = 853.6 \text{ dollars}^2.$$

In the expression for  $E(X^2 | I)$ , we have to account for the fact T (and thus also X) is a mixed random variable. When I=0, T is continuous over the interval [0, 60) but discrete at the point 60 since  $\Pr[T=60|I=0]=(1-.02)^{60}>0$ . Likewise, when I=1, T is continuous over the interval [0, 120) but discrete at 120 months.

In the following calculations, I round to one place to the right of the decimal at each step, which introduces some minor error.

$$\sigma^2(X | I = 0) = E(X^2 | I = 0) - E(X | I = 0)^2 = 463.7 \text{ dollars}^2$$

$$\sigma^2(X | I = 1) = E(X^2 | I = 1) - E(X | I = 1)^2 = 342.8 \text{ dollars}^2$$

$$E_I(\sigma^2(X | I)) = .5 \cdot 463.7 + .5 \cdot 342.8 = 403.3 \text{ dollars}^2$$

$$\sigma_I^2(E(X | I)) = \frac{(34.8 - 28.7)^2}{2} + \frac{(22.6 - 28.7)^2}{2} = 37.2 \text{ dollars}^2$$

$$\sigma^2(X) = E_I(\sigma^2(X | I)) + \sigma_I^2(E(X | I)) = 403.3 + 37.2 = 440.5$$

$$\sigma(X) | E(X) = \frac{\sqrt{440.5}}{28.7} = 0.7$$

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