Take the Money and Run

THIS ISSUE'S PUZZLE

Once upon a time, there were four thieves who successfully trekked across a desert, rowed across a river, traversed a forest, tunneled under the wall, and made off with 18 sacks of the Sultan's jewels and gold. The names of these thieves were Antone, Bruno, Cristoph, and Dan. Because Antone had organized the raid, they all agreed his share should be the largest, so he got eight sacks. Bruno was the second-in-command, so he got five sacks. Cristoph got three and Dan got the last two. Everyone agreed to this division, and swore an oath to abide by it.

They seemed to be set for life, because even two sacks of gold and jewels will go pretty far in these inflationary times. They made it back through the tunnel and crossed the forest without any difficulty but ran into some trouble at the river's edge.

You see, as is common for these types of stories the boat was only large enough to carry across two people at a time. Since the thieves were traveling with heavy sacks of gold and jewels, they were further restricted because they couldn't fit two people and even a single sack of gold and jewels into the boat at one time. They could either carry up to two people, or one person plus some of the Sultan's treasure.

Although they had all agreed to the division of treasure and all trusted each other enough to undertake this huge expedition, they also had a fair understanding of human psychology. They knew that if, at any point, any members of the gang were on the homeward side of the river with more than their share of the treasure, they would likely succumb to the temptation to abandon the rest of the gang and run off. None of them wanted to place his companions in such a predicament.

The gang needed a way to get all of themselves and all the treasure across the river, so that each trip took one thief, two thieves, or one thief plus some amount of treasure. Further, at no point could any subset of the gang of thieves be on the homeward side of the river in possession of more than their agreed-upon share of the treasure. (Having some thieves on the homeward side of the river with less than their share of the treasure is fine.) Finally, since they are in a bit of a hurry the thieves need to minimize the total number of trips across the river. What's the minimum number of trips required, and who and/or what is on the boat for each trip?

PREVIOUS ISSUE'S PUZZLE

Coin Tricks

I spent the weekend visiting an old friend, Mr. Maxwell Chance. He's an eccentric sort, mostly harmless, but he had recently developed a numismatic obsession and so playing with coins took up most of our time. This lead to the following three "quickie" puzzles:

1. The Scales: Max set up a coin slot that led down to a pair of balance scales. The arms of the scales were slanted so that a coin might roll either to the right or left, then fall into a bucket. He took $100 worth of dimes and set one dime in each bucket. He explained that the angle of the scale's arms was such that the probability of a coin rolling right or left was proportional to the number of coins already in each bucket. For example, if the right bucket had 13 coins and the left bucket had four, the probability of the next coin rolling to the right was 13/17.

To start off, Max placed a single dime in each bucket (to prevent the first roll from completely deciding the remaining outcome).

"What would you give me now in advance," he asked, "for the right to the lighter bucket after I roll the remaining 998 dimes into the scales?"

Dimes spend as well as any other cash, so what is a fair bid?

2. The Chessboard: Max set out quarters down the diagonal of a chessboard, then played a (somewhat tedious) game of laying out additional coins based on the rule that if an empty square shared two or more sides with a filled square, it should also be filled. The board quickly filled in.

"The diagonal is kind of a special case," he explained. "If I lay them out at random, I usually need nine or 10 to be sure of filling in the board. I think eight is the minimum, but I haven't been able to prove it."

Can you?

3. The Table: Max took $2.82 in nickels and pennies, and threw them onto a table that he had covered with a large sheet of paper. After carefully checking that no three coins fell onto a single line, he challenged me to connect the coins by straight line segments, such that each line connected a nickel to a penny and no two lines crossed.

Easy enough—but your puzzle is to find an algorithm that always works.

Solution

All three puzzles are ones that can be dealt with quickly, but also not so quickly. Proofs by induction seemed to come to mind for a lot of solvers. The quick solutions are:

1. While not obvious, the split of coins between the two buckets is uniform—there's an equal chance of the lighter bucket having anywhere from one to 999 coins. Thus, the fair value of the lighter bucket is $25.025

2. Note that the sum of the perimeters of the covered squares stays constant. That is, each time a new square is covered, two edges that were previously outer edges become interior and two new exterior edges are created. Thus, to eventually get the outer perimeter of the entire board, you need to cover at least eight squares. Moreover, those eight squares cannot have any edges in common. Of course, this doesn't show that any set of eight squares will eventually cover the board, but it does show that the minimum is eight.

3. Although I didn't make this clear enough, my intention was that there would be an equal number—47—of pennies and nickels. To attach each nickel to a penny without any lines crossing, it's sufficient to minimize the total length of the edges. If two lines did cross, say vx and yz, they then would be cross diagonals of a convex quadrilateral. So, by the triangle inequality, the sides vz and yx would be short. ●

SOLVER LIST

Solvers of all three puzzles*—Robert Bartholomew, Bob Byrne, Bill Cross, Mark Evans, Rui Guo, John Hubenschmidt, Lee Michelson, Geoff Moak, David Promislow, Noam Segal, Al Spooner, Kevin Trapp*

Solutions may be e-mailed to cont.puzzles@gmail.com or mailed to Puzzles, 65 W. 35th Place, Eugene, Ore. 97405.

In order to make the solver list, please make sure that your answers and solutions are received by Sept. 30, 2008. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

Solvers of at least 1 puzzle*—Jason Choi, John Cook, Mark Fowler, Philip Silverman, Barry Zurbuchen*

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