Applications of Monte Carlo Methods to Finance and Insurance

Thomas N. Herzog and Graham Lord

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AS SOMEONE WHO MAJORED IN APPLIED MATHEMATICS, I considered myself to be quite knowledgeable about the mathematics underlying Monte Carlo methods—until I picked up this excellent book.

I studied the subject about 30 years ago, before powerful computers became cheap and ubiquitous. Techniques that were only theoretically possible then are now commonplace, and the mathematical theory has advanced, too. This book, while certainly formidable, describes the current state of the art in something approaching ordinary English (along with lots of formulas). It also gives numerous examples of difficult practical problems to which Monte Carlo techniques have been applied, with great success.

Tom Herzog and Graham Lord are two award-winning mathematicians who are pioneers in this field. Dr. Herzog is chief actuary at the U.S. Department of Housing and Urban Development, where he has used Monte Carlo methods to analyze government-insured mortgages. The financial experience on blocks of mortgages is quite difficult to model because, among other things, the repayment rate depends on economic experience, such as changes in interest rates. Predicting future interest rates is, of course, a challenge in itself; applying those predictions to mortgage experience adds another major level of complexity.

Dr. Lord, who is now affiliated with Princeton University, has used Monte Carlo techniques to advise the U.S. Department of Energy on how to draw down the Strategic Petroleum Reserve in an optimal way. The complexity of that problem is self-evident.

The book begins with a subject that most readers probably have never considered: generation of random numbers. While many of us know how to get Excel (or any of a multitude of similar programs) to generate a random number or string of them, few of us have thought about what the software is doing to get these numbers, whether they're truly random or what *random* means, anyway.

The authors devote three entire chapters (more than 80 pages) to describing countless random-number generators, going back to pre-computer times. It turns out that most of these algorithms are actually deterministic, making their output, in technical terms, pseudo-random or quasi-random sequences of numbers.

Congruential random-number generators can produce sequences of numbers that superficially appear to be random but that exhibit surprising patterns when overlapping pairs are plotted in multi-dimensional space. The linear congruential generator, "RANDU," which was developed by IBM in the 1960s and used in early computers, fails a three-dimensional test of randomness. According to the authors, it should never have been used. The multiplicative congruential generator that replaced RANDU, called "GGL," avoids this problem until one reaches 24 dimensions.

If one is going to use Monte Carlo simulation, the choice of random-number generator is more important than it may seem to be. As the book says, "It is not always a good idea to turn on your favorite vendor-supplied random-number generator and to turn off your mind.… Random numbers are too important to be left to chance."

While many problems require the use of random numbers taken from the uniform distribution on [0,1), the book also describes how random observations can be generated from various probability distributions that are known to correspond closely to real-life problems. These distributions can be discrete or continuous. One of the most familiar to actuaries is the standard normal distribution. The book also considers the much more complex multivariate normal distributions with covariance.

The middle part of the book describes numerous variance-reduction techniques and measures of fit. In doing so, it introduces such concepts as stratified sampling, discrepancy, and the bootstrap method (which uses a small sample to represent a much larger universe). Interestingly, equally spaced observations have a lower discrepancy than random values. The authors go on to describe a number of specialized sequences that produce superior results under various circumstances.

The last part of the book provides examples of actual problems that have been analyzed using Monte Carlo techniques. One of these is generation of interest-rate scenarios, which has obvious applications to insurance. Another is modeling of the financial experience of home-equity conversion mortgages, which allow elderly homeowners to access methodically the substantial equity that may exist in their homes. The payout available under such a "reverse" mortgage depends on whether the homeowner shares future price appreciation with the mortgage company, as well as many more obvious variables, including mortality.

While this book is hardly light reading, it's quite interesting and informative. I highly recommend it. ●

—Bruce D. Schobel