Blueland Soccer Tournament

This Issue's Puzzles

The country of Blueland has four soccer teams that compete for the national championship. The teams' names correspond to their professions—Miners, Bankers, Police, and Army. (Blueland is a small nation and does not have professional players—all of them have other jobs.) The tournament has two stages. Stage 1 is a round robin in which each team plays against all of the others. Stage 2 is the final match between the two best teams from Stage 1, and it decides the championship. In the event two or more teams have the same number of points at the end of the round-robin stage, a random draw breaks the tie. In the middle 1990s, the country followed the rest of the world and changed the scoring system so that a win is worth three points (instead of two points under the old system). A tie and a loss bring one and zero points, respectively, before and after the change. All four teams are equally strong, and the probability of a tie in any given game is p. The Miners and the Bankers, however, are less than honest and never play competitive games between each other. They always agree on the result before the game begins. Given their records, the probability of a tie in the game between the Miners and the Bankers is q. Answer the following questions:

1. If there's a tie between the Miners and the Bankers, what probability p maximizes chances for the Police to make the final game?
2. If the Miners beat the Bankers, what probability p minimizes chances for the Army to make the final game?
3. Assuming uniform distribution of p on the interval [0, 1], what probability q triggers the expectation of the Police winning the championship to be equal to ½?
4. How do the answers to the previous three questions change if the old scoring system is restored?

Previous Issue's Puzzles

Chess Puzzle

White to move and mate in three.

Initial position:

- White—Kg8, Rg3, Qg4, Bf4, pawns c2, d6
- Black—Ke4, Bb8, Na1, pawns b6, c6.

Actuarial Exams and Mean CEO

ANSWERS:

1) Skip 2,043 candidates and remember the one with the highest number of exams passed (X). Then hire the next one with the number of exams passed greater than X. Surely, if there is no such person, Mr. Purple will get fired.

2) 63.21 percent, very close (but not exactly equal) to 1 - 1/e.

3) The probability of getting fired will start at 50 percent for three candidates, then goes down to 45.83 percent for four candidates, then keep going down and approaches 1 - 1/e. Since we talk discrete mathematics here, there is no 1 - 1/e limit per se.

This puzzle is an analog to a better-known Sultan Dowry puzzle. (In the Sultan puzzle there are 100 daughters with different dowries. Each daughter is shown to a commoner who can only marry the daughter with the highest dowry.)

Since no actual figures are given in the puzzle, the only useful information Mr. Purple has is the number of actuaries he has interviewed, the number of candidates to be interviewed, and the highest number of exams passed among those whom he has interviewed. It is evident that Mr. Purple will hire an actuary with a better exam "collection." If the hiring manager selects an actuary too soon, there is a pretty substantial probability of missing a better candidate he has yet to see. On the other hand, if he selects too late, there is a substantial probability that he has already missed the best actuary. Therefore, he will wait a certain number of candidates, remember the best performance he has seen so far, and then hire the next one with a higher number of exams passed.

Assume the number of candidates is N and the number of actuaries that Mr. Purple skips is M.

The probability that person (M+1) is to be hired is 1/N.

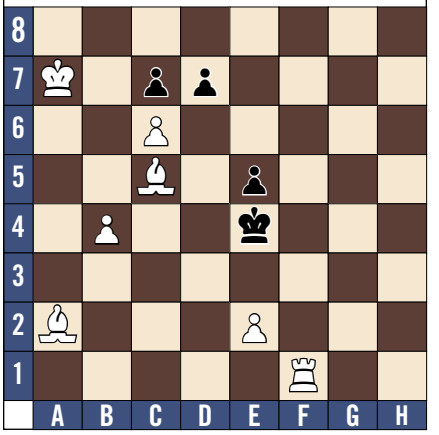
The probability that person (M+2) is to be hired is 1/N * M/(M+1). (Hint: to be hired, candidate M+2 must pass more exams than M and M+1 and person M+1 cannot pass more exams than actuary M.)

Reasoning the same way, we go on to the probability of the person N to be hired as 1/N * M/(N-1).

Therefore, the total probability that Mr. Purple actually achieves his goal is the sum M/N * (1/M + 1/(M+1) + ... + 1/(N-1)). This sum asymptotically be-

CHESS PUZZLE

White to Move and Mate in Three



haves like a logarithm. The answer to the puzzle could be obtained simply by calculating the sum and finding such M that maximizes that probability of success. Since it is not required to solve the problem mathematically, I will not show it here but one might mathematically prove that such M lies near N/e, and the sum lies near 1/e.

Since the puzzle disallows ties between the number of exams, then the correct strategy is slightly different: skip 2,043 candidates and remember the one with the highest number of exams passed (X). Then hire the next one with the number of exams passed greater than max (X, 5554) because we know there should be at least one person with 5,555 exams or more. Technically speaking, the actual resulting probability of being fired because of this new wrinkle is less than 63.21 percent.

Solutions may be e-mailed to
cont_puzzles @yahoo.com
or mailed to
Puzzles, 25 Sparrow Walk,
Newtown, Pa. 18940.

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by Sept. 30, 2005. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

Chess Puzzle

► Case A

1. Rh3!—any response except c5, Kd5, Kd4, Bxd6
2. Bh2+ Kd5
3. Rh5#

► Case B1

1. Rh3! c5
2. c4—any response except Bxd6
3. Bh2#

► Case B2

1. Rh3! c5
2. c4 Bxd6
3. Bxd6#

► Case C

1. Rh3! Kd5
2. Rh5+ —any response
3. Bd2#

► Case D

1. Rh3! Kd4
2. Bh2+ - any response
3. Rh5#

► Case E

1. Rh3! Bxd6
2. Bxd6+ Kd5
3. Rd3#

Solver lists

Due to an administrative deadline, names of only those people who submitted correct solutions by Aug. 13, 2005, are shown on the lists.

EXAMS PUZZLE: *Steven Berman, Bob Byrne, Mary Campbell, Bill Carroll, Ning Chen, John Cook, Mike Failor, Steven Gallancy, Charles Greeley, Len Helfgott, Stuart Klugman, Jack Krull, Lee Michelson, David Promislow, Al Spooner, Thomas Struppeck, Tony Torelli, Jim Walsh, Cliff Woodhall*

CHESS PUZZLE: *Mike Crooks, Lee Michelson, David Raheb, Edward Scher, Steven Shawcross, Tim Swankey*