The Fully Taxed Life Annuity:
A Benchmark for Evaluating Tax Deferred Retirement Saving

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(Editor's note: This online "Workshop" is expanded from the one in the print version of the September/October 2003 Contingencies.*)*

These days everyone is encouraged to save for his or her retirement in some sort of tax-deferred saving plan such as a 401(k). Taxes on investment income and, in most cases, on contributions are deferred until retirement. This is touted as a good deal, specially by those who want to invest your savings. A question that is hard to quantify, however, is how much of a good deal? Normally, such a question is answered by comparing the present value of a tax-deferred investment to the present value of an equivalent fully taxed investment. While the mathematics can get fairly complex, an answer is usually possible because one is dealing with certain, not contingent, cash flows. Evaluating the present value of retirement savings and withdrawals is complicated, however, by the fact a retiree does not know how long she will live, i.e., the cash flow or annuity in retirement is contingent on being alive. And, while the actuarial present value of a life annuity is easy to calculate, the actuarial present value of a taxable life annuity is not. As noted below, it is not enough to simply replace the interest rate used in the actuarial calculations with an after-tax interest rate. This paper develops the mathematics of a fully taxable life annuity that can be used as a benchmark to evaluate the benefits of tax-deferred saving and life annuities.[2]

Life annuities may be calculated with the basic actuarial commutation functions D and N that incorporate interest and mortality rate assumptions. It is possible to incorporate a tax rate assumption

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also and develop new commutation functions, DT and NT, for calculations involving fully taxed annuities.

The Mathematics

The actuarial present value of 1 to be received in one year by a person age x is

(1) $${}_1E_x = v \cdot p_x$$

where $v = \frac{1}{1+i}$, i is the interest rate, $p_x = \frac{l_{x+1}}{l_x}$ is the probability of living from age x to age $(x+1)$, and l_x is the number of persons living at age x. Expressed in terms of commutation functions, (1) can be represented as

(2) $${}_1E_x = \frac{D_{x+1}}{D_x}$$

where $D_x = v^x \cdot l_x$.

Inverting (2) and solving for 1 gives

(3) $$1 = {}_1E_x \cdot \frac{D_x}{D_{x+1}}$$

The income earned in one year from the investment of ${}_1E_x$ is

(4) $$1 - {}_1E_x$$

Substituting for 1 from (3), the income is

(5) $${}_1E_x \cdot \frac{D_x}{D_{x+1}} - {}_1E_x$$

The after-tax value of the investment at year $(x+1)$, call it S_t, where t is the tax rate is therefore

[2] For a study dealing with certain, not contingent, annuities see: U.S. Library of Congress, Congressional Research Service, *IRA Alternatives: A Comparison of Taxpayer Benefits*, Report No. 90-191 E by Donald W. Kiefer, April 6, 1990, 20 p.

(6) $$S_t = \left({}_1E_x \cdot \frac{D_x}{D_{x+1}} - {}_1E_x \right)(1-t) + {}_1E_x$$

Simplifying (6).

(7) $$S_t = {}_1E_x \left(\frac{(1-t)\,D_x}{D_{x+1}} + t \right)$$

For subsequent calculations, let

(8) $$\theta_x = \frac{(1-t)\,D_x}{D_{x+1}} + t$$

Then solving (7) for the actuarial present value of S_t gives

(9) $${}_1E_x = \frac{S_t}{\theta_x}$$

S_t may be considered a constant while θ_x is age specific. θ_x represents the after-tax value at age $(x+1)$, if living, of 1 invested at age x. $1/\theta_x$ therefore acts as a discount factor; it discounts the after-tax value of S, i.e., S_t, at age $(x+1)$ to its actuarial present value at age x.

If S_t were to be received at age $(x+n)$ instead of age $(x+1)$, its actuarial present value would be, by extension

(10) $${}_nE_x = \frac{S_t}{\theta_x \cdot \theta_{x+1} \cdots \theta_{x+n-1}}$$

It is now possible to define the commutation function DT.

(11) $$DT_x = \prod_{j=x}^{\omega} \theta_j$$

where θ_ω is normalized to $(1-t)\,D_\omega + t$.[3]

Using the new commutation function, (9) may now be expressed as

(12) $${}_1E_x = S_t \cdot \frac{DT_{x+1}}{DT_x}$$

And, the actuarial present value of an after-tax amount S_t to be received at age $(x+n)$ is

(13) $${}_nE_x = S_t \cdot \frac{DT_{x+n}}{DT_x}$$

For after-tax annuities paid over many years the commutation function NT can be formed by summing (11).

(14) $$NT_x = \sum_{j=x}^{\omega} DT_j$$

With this function the actuarial present value at age x of an after-tax amount S_t to be received annually for life beginning at age $(x+1)$ is

(15) $$a_x = S_t \cdot \frac{NT_{x+1}}{DT_x}$$

In summary, the commutation functions DT and NT perform the same functions as D and N in the calculation of life annuities, but they operate on after-tax annuities instead of before-tax annuities. To clarify the relationship between a_x and S_t, a_x is the amount that must be invested at age x in order to receive an annual amount up to age $(x+n)$ (if living) which, after the annual payment of taxes on the investment income, is worth S_t.

This derivation of DT and NT involves the taxation of the annual investment income, which includes both interest and survivorship income. Survivorship is that part of a life annuity attributable not to past contributions or interest income of the annuitant, but to the annuitant's share of the remaining assets of those annuitants who died in the past year. Survivorship represents an increasing part of a life annuity as the annuitant ages. Under the U.S. tax code, only after-tax contributions made to a tax deferred savings plan may be deducted from the annuity in determining taxable income.[4] This means that both the interest income and survivorship are subject to taxation in retirement, not just the interest income. Tax deferred or before-tax contributions to a tax deferred savings plan cannot be deducted from an annuity and are therefore subject to taxation in retirement also.

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[3] θ_ω, where ω is the last age in the mortality table, can be normalized to any number since DT_x is the product of θ_j s and all actuarial calculations are in terms of ratios of the commutation functions.

[4] See 26 U.S.C. §72.

If just an after-tax interest rate were used in normal actuarial calculations to compute "after-tax" life annuities, survivorship would not be taxed. This would give incorrect values in all calculations dealing with fully taxed life annuities.

An Example

There are several ways to quantify the benefits of tax-deferred annuities. First, there is the difference in the annual after-tax cost of the fully taxed annuity versus the tax-deferred annuity. Second, there is the difference in taxes paid on the two types of annuities. And third, there is the difference in effective tax rates.

Consider a female age 25 who wants to provide for a $10,000 after-tax life annuity beginning at age 65 and payable at the end of the year. Yearly contributions are placed in a qualified 401(k) plan invested in a deferred life annuity[5]. Assume an annual interest rate of 6%, a tax rate of 28%, and mortality as reflected in Table 886, 1996 US Annuity 2000 Female, from the Society of Actuaries' Table Manager software. What is the annual cost of this life annuity compared to a fully taxed annuity? What is the difference in taxes paid? What is the difference in effective tax rates?

After-tax cost

Since the contributions and investment income are tax deferred during the working years, the entire annuity is taxable during the retirement years. Thus, to have $10,000 after-tax, the annuity would have to provide $13,888.89 before-tax $(10{,}000/(1-0.28))$.

[5] In reality the contributions would most likely not be invested in a deferred life annuity. If the investor died before the end of the deferral period there would be nothing left for her estate. The contributions would more likely be invested in mutual funds or other non-contingent investments and only annuitized on retirement. Assuming otherwise, as this paper does, simplifies the calculations although the numerical results are very close to those for non-contingent investments.

The annual contribution for this annuity is

(16) $$\frac{10{,}000}{(1-0.28)} \cdot \frac{N_{66}}{N_{25} - N_{66}} = 913.23$$

Since the contribution is tax deductible, the annual after-tax cost of this annuity is $657.52. (The values of the commutation functions are presented in the appendix.)

The after-tax cost of a fully taxed deferred life annuity is calculated in a similar manner, except that the taxes are accounted for in the commutation functions. The after-tax cost of a similar life annuity where taxes on the investment income are paid annually during the accumulation and withdrawal periods is

(17) $$10{,}000 \cdot \frac{NT_{66}}{NT_{25} - NT_{66}} = 1{,}280.83$$

The benefit of saving in a tax-deferred plan is obvious. In this example, the benefit is having an extra $623.31 a year (1,280.83 – 657.52) to invest or consume from age 25 to 65 while maintaining the same after-tax income in retirement.

Taxes

Another way to measure the benefit of a qualified annuity versus a fully taxable one is to compare the taxes paid over the life of the annuities. For the qualified annuity, the actuarial present value at age 25 of the taxes paid on the withdrawals during retirement is

(18) $$\left(\frac{10{,}000}{1-0.28} - 10{,}000\right) \cdot \frac{N_{66}}{D_{25}} = 4{,}069.65$$

There are no taxes paid, of course, on the income from which the contributions were made or on the investment income during the accumulation period.

For the fully taxed annuity computing the actuarial present value of the taxes paid requires a year-by-year calculation of the investment income earned; it will not be presented here. The actuarial present value of the taxes paid, however, is $17,847.86; this is 338.6 percent greater than the taxes paid on the qualified annuity. (The $17,847.86 includes the taxes paid on the income from which the contributions were made.)

The difference in the actuarial present value of taxes paid on the qualified annuity versus the fully taxed annuity is $13,778.21. This represents a pure loss to the taxing authority. To maintain government programs at their current level borrowing must be increased, other taxes imposed or increased, or some combination of the two.

Effective tax rates

The loss in taxes is reflected in the effective tax rate on the qualified annuity. The tax rate on the fully taxed annuity, as stipulated in the example, is 28 percent, on the income from which contributions were made and on the annual investment income. To calculate the effective tax rate on the qualified annuity we find that tax rate which, when applied to the annual contributions and investment income during the accumulation period, would allow the fund to grow by retirement age to the same amount as the fully taxed annuity. The value of the fund or reserve of the fully taxed annuity at age 65 is

(19) $$10,000 \cdot \frac{NT_{66}}{DT_{65}} = 143,798.97$$

The effective tax rate on the qualified annuity's annual contribution of $913.23 and annual investment income that allows it to reach $143,798.97 at age 65 is 3.90 percent, or 86 percent less than the 28 percent tax rate on the fully tax annuity. This means that the deferred taxes paid during retirement at a 28 percent tax rate on the contributions and investment income from the accumulation period are equivalent to paying taxes at only a 3.90 percent rate during the accumulation period.

Conclusion

This paper demonstrates the possibility of quantifying the benefits of tax-deferred life annuities using the fully taxed annuity as a benchmark. The mathematics of the fully taxed life annuity is presented in the commutation functions developed herein. This makes it possible to compare the after-tax cost, taxes paid, and effective tax rates on tax deferred annuities compared to equivalent after-tax annuities. In the example the benefits of investing in a qualified as opposed to a fully taxed annuity are quite obvious. It is possible to vary the assumptions, e.g., different interest and tax rates during the accumulation and withdrawal periods, contributions increasing with age as salary increases, incorporating inflation expectations, etc., and quantify the benefits of more complex situations. It is also possible to quantify the benefits of non-qualified annuities (Traditional IRAs) and Roth IRAs.

Appendix

The following routines compute the *D, N, DT* and *NT* commutation functions for the calculations used in this paper. Table 886, 1996 US Annuity 2000 Female, from the Society of Actuaries' Table Manager software was used with an interest rate of 6% and a tax rate of 28%. The programming and calculations were done with Derive™ 5.06: The Mathematical Assistant for Your PC.

```
#20:  InputMode := Word

#21:  Precision := Approximate

#22:  PrecisionDigits := 20

#23:  NotationDigits := 15

#24:  Notation := Decimal

      tab(l, r, n_, m_, i) :=
        Prog
          n_ := DIM(l)
          m_ := [r]
          i := 1
#25:      Loop
            If i = n_
               RETURN m_
            m_ := APPEND(m_, [m_↓i·(1 - l↓i)])
            i :+ 1

      Annuity_Tables(l, i, t := 0, r := 10^9, m, l_, k, dx, nx, w, dtx, ntx) :=
        Prog
          m := ["Age", "l", "D", "N", "DT", "NT"]
          l_ := tab(l↓↓2, r)
          k := DIM(l_)
          age := l↓↓1
          α := age↓1
#26:      ω := age↓k
          dx := VECTOR(l_↓n_·(1 + i)^(- (n_ + α - 1)), n_, k)
          nx := VECTOR(Σ(dx↓i_, i_, n_, k), n_, k)
          w := VECTOR(dx↓n_·(1 - t)/IF(n_ < k, dx↓(n_ + 1), 1, 1) + t, n_, k)
          dtx := VECTOR(Π(w↓i_, i_, n_, k), n_, k)
          ntx := VECTOR(Σ(dtx↓i_, i_, n_, k), n_, k)
          tables := APPEND([m], [age, l_, dx, nx, dtx, ntx]`)
          "Tables Done!"

      D(x) :=
        If x > ω
           0
#27:       If x < α
              "Invalid age"
              tables↓(x - α + 2)↓3

      N(x) :=
        If x > ω
           0
#28:       If x < α
              "Invalid age"
              tables↓(x - α + 2)↓4

      DT(x) :=
        If x > ω
           0
#29:       If x < α
              "Invalid age"
              tables↓(x - α + 2)↓5

      NT(x) :=
        If x > ω
           0
#30:       If x < α
              "Invalid age"
              tables↓(x - α + 2)↓6
```

Appendix

The following is the mortality table giving the age and probability of dying at that age.

0	0.001615
1	0.00068
2	0.000353
3	0.000261
4	0.000209
5	0.000171
6	0.000141
7	0.000118
8	0.000118
9	0.000121
10	0.000126
11	0.000133
12	0.000142
13	0.000152
14	0.000164
15	0.000177
16	0.00019
17	0.000204
18	0.000219
19	0.000234
20	0.00025
21	0.000265
22	0.000281
23	0.000298
24	0.000314
25	0.000331
26	0.000347
27	0.000362
28	0.000376
29	0.000389
30	0.000402
31	0.000414
32	0.000425
33	0.000436
34	0.000449

#31: a2000f :=

35	0.000463
36	0.000481
37	0.000504
38	0.000532
39	0.000567
40	0.000609
41	0.000658
42	0.000715
43	0.000781
44	0.000855
45	0.000939
46	0.001035
47	0.001141
48	0.001261
49	0.001393
50	0.001538
51	0.001695
52	0.001864
53	0.002047
54	0.002244
55	0.002457
56	0.002689
57	0.002942
58	0.003218
59	0.003523
60	0.003863
61	0.004242
62	0.004668
63	0.005144
64	0.005671
65	0.00625
66	0.006878
67	0.007555
68	0.008287
69	0.009102
70	0.010034

71	0.011117
72	0.012386
73	0.013871
74	0.015592
75	0.017564
76	0.019805
77	0.022328
78	0.025158
79	0.028341
80	0.031933
81	0.035985
82	0.040552
83	0.04569
84	0.051456
85	0.057913
86	0.065119
87	0.073136
88	0.081991
89	0.091577
90	0.101758
91	0.112395
92	0.123349
93	0.134486
94	0.145689
95	0.156846
96	0.167841
97	0.178563
98	0.189604
99	0.201557
100	0.215013
101	0.230565
102	0.248805
103	0.270326
104	0.295719
105	0.325576
106	0.360491
107	0.401054

108	0.44786
109	0.501498
110	0.562563
111	0.631645
112	0.709338
113	0.796233
114	0.892923
115	1

The following matrix shows the values of the *D, N, DT,* and *NT* commutation functions for each age in the mortality table.

```
#32:  Annuity_Tables(a2000f, 0.06, 0.28)
```

Age	l	D	N	DT	NT
0	1000000000	1000000000	17428297160.4761	9971874.48754777	233474206.082169
1	998385000	941872641.509433	16428297160.4761	9547629.74350114	223502331.594621
2	997706098.2	887954875.578497	15486424518.9667	9147698.50179285	213954701.85112
3	997353907.947335	837397573.120206	14598469643.3882	8766617.96083298	204807003.349327
4	997093598.577361	789791521.088322	13761072070.268	8401978.42228557	196040385.388494
5	996885206.015258	744930617.604165	12971280549.1796	8052812.17346981	187638406.966208
6	996714738.645029	702644560.819391	12226349931.5755	7718371.06805491	179585594.792739
7	996574201.86688	662778762.204071	11523705370.7561	7397982.0265191	171867223.724684
8	996456606.11106	625189202.179369	10860926608.552	7091011.64845172	164469241.698165
9	996339024.231539	589731537.597652	10235737406.3726	6796778.63750324	157378230.049713
10	996218467.209607	556283188.756229	9646005868.77503	6514740.17128557	150581451.41221
11	996092943.682739	524729336.862685	9089722680.0188	6244382.30045202	144066711.240924
12	995960463.321229	494961837.604606	8564993343.15612	5985213.46525643	137822328.940472
13	995819036.935437	466878823.607232	8070031505.55151	5736763.47647641	131837115.475216
14	995667672.441823	440384771.722683	7603152681.94428	5498586.56851259	126100351.998739
15	995504382.943543	415389196.811434	7162767910.22159	5270251.9270359	120601765.430227
16	995328178.667762	391807238.607168	6747378713.41016	5051351.07113103	115331513.503191
17	995139066.313815	369559240.784748	6355571474.80299	4841496.24427503	110280162.43206
18	994936057.944287	348569670.471347	5986012234.01824	4640312.1488412	105438666.187785
19	994718166.947597	328767295.956145	5637442563.54689	4447439.26136514	100798354.038943
20	994485402.896531	310085249.44235	5308675267.59075	4262536.2748799	96350914.7775787
21	994236781.545807	292460120.877349	4998590018.1484	4085272.82391569	92088378.5026988
22	993973308.798697	275832659.382374	4706129897.27105	3915338.13718092	88003105.6787831
23	993694002.298925	260146368.306686	4430297237.88867	3752428.26708246	84087767.5416021

#33:

24	993397881.48624	245347966.687671	4170150869.58199	3596252.02715493	80335339.2745197
25	993085954.551453	231387667.383142	3924802902.89432	3446535.49024032	76739087.2473647
26	992757243.100497	218217998.174753	3693415235.51117	3303010.74580998	73292551.7571244
27	992412756.337141	205794600.499421	3475197237.33642	3165425.76097404	69989541.0113144
28	992053502.919347	194075568.730227	3269402636.837	3033538.49339557	66824115.2503404
29	991680490.802249	183021317.279608	3075327068.10677	2907116.51087837	63790576.7569448
30	991294727.091327	172594454.704892	2892305750.82716	2785936.62459048	60883460.2460664
31	990896226.611036	162759501.635944	2719711296.12227	2669782.58128331	58097523.621476
32	990485995.573219	153483131.322893	2556951794.48633	2558448.87527634	55427741.0401926
33	990065039.025101	144733868.860454	2403468663.16343	2451738.2010643	52869292.1649163
34	989633370.668086	136481853.673236	2258734794.30298	2349459.41834205	50417553.963852
35	989189025.284656	128698654.076356	2122252940.62974	2251425.95890986	48068094.5455099
36	988731030.765949	121357609.999546	1993554286.55338	2157460.92976018	45816668.5866001
37	988255451.14015	114433242.442581	1872196676.55384	2067390.36348458	43659207.6568399
38	987757370.392776	107901479.32867	1757763434.11126	1981046.7446875	41591817.2933553
39	987231883.471727	101739694.095912	1649861954.78259	1898270.31649316	39610770.5486678
40	986672122.993798	95926422.348453	1548122260.68667	1818906.02640382	37712500.2321746
41	986071239.670895	90441512.4124932	1452195838.33822	1742806.26529726	35893594.2057708
42	985422404.795192	85266039.525779	1361754325.92573	1669830.47195249	34150787.9404736
43	984717827.775763	80382145.5731303	1276488286.39995	1599843.58881473	32480957.4685211
44	983948763.15227	75772987.8466393	1196106140.82682	1532715.95599801	30881113.8797063
45	983107486.959775	71422832.0207834	1120333152.98018	1468325.34599642	29348397.9237083
46	982184349.02952	67316760.3599207	1048910320.9594	1406553.29705655	27880072.5777119
47	981167788.228274	63440648.5971209	981593560.599479	1347285.23548249	26473519.2806553
48	980048275.781906	59781380.0161053	918152912.002358	1290414.34490387	25126234.0451728
49	978812434.906145	56326411.0338726	858371531.986253	1235835.39223086	23835819.7002689
50	977448949.184321	53064102.2106626	802045120.952381	1183450.40886403	22599984.3080381
51	975945632.700475	49983480.7749648	748981018.741718	1133165.49897016	21416533.899174
52	974291404.853048	47074300.7311804	698997537.966753	1084892.32491067	20283368.4002039
53	972475325.674402	44326937.9571863	651923237.235573	1038546.8980196	19198476.0752932
54	970484668.682746	41732264.825649	607596299.278386	994047.872889367	18159929.1772736
55	968306901.086222	39281714.7390379	565864034.452737	951318.023008838	17165881.3043842
56	965927771.030253	36967169.4018152	526582319.713699	910282.661700776	16214563.2813754
57	963330391.253953	34780910.078579	489615150.311884	870869.062843423	15304280.6196746
58	960496273.242884	32715645.8878564	454834240.233305	833007.250875689	14433411.5568312
59	957405396.235588	30764497.1126314	422118594.345449	796630.012508475	13600404.3059555

60	954032457.02465	28920862.0653807	391354097.232817	761670.655076113	12803774.293447
61	950347029.643164	27178434.6936058	362433235.167436	728063.481518121	12042103.6383709
62	946315657.543418	25531267.7109769	335254800.47383	695745.221024967	11314040.1568528
63	941898256.054005	23973667.6917944	309723532.762853	664653.192716862	10618294.9358278
64	937053131.424863	22500327.4954602	285749865.071059	634728.171370125	9953641.743111
65	931739103.116553	21106347.3002202	263249537.575599	605915.211422737	9318913.57174088
66	925915733.722074	19787200.5939565	242143190.275378	578163.369678184	8712998.36031814
67	919547285.305534	18538777.5738408	222355989.681422	551427.072790837	8134834.99063995
68	912600105.565051	17357280.2917645	203817212.107581	525664.332423995	7583407.91784912
69	905037388.490233	16239094.8207422	186459931.815816	500834.229115287	7057743.58542512
70	896799738.180195	15180459.0374375	170220836.995074	476889.395321338	6556909.35630983
71	887801249.607295	14177488.9730716	155040377.957637	453776.054475896	6080019.9609885
72	877931563.115411	13226299.8378848	140862888.984565	431436.334204319	5626243.9065126
73	867057502.774663	12323093.2906536	127636589.14668	409810.024654109	5194807.57230828
74	855030548.153676	11464301.5694518	115313495.856027	388837.950288053	4784997.54765417
75	841698911.846864	10646745.4522461	103849194.286575	368466.088456192	4396159.59736612
76	826915312.159185	9867684.9199273	93202448.8343289	348647.402330731	4027693.50890992
77	810538254.401873	9124769.26423409	83334763.9144016	329341.228311974	3679046.10657919
78	792440556.257588	8416067.37368139	74209994.6501675	310514.748747155	3349704.87826722
79	772504336.743259	7739939.57612671	65793927.2764861	292140.265376871	3039190.12952006
80	750610791.335618	7094888.63075444	58053987.7003594	274191.486940773	2747049.86414319
81	726641536.935898	6479554.29444203	50959099.069605	256643.444033573	2472858.37720242
82	700493341.22926	5892818.42750617	44479544.7751629	239475.689079808	2216214.93316884
83	672086935.255731	5333823.44776786	38586726.3476568	222673.340433858	1976739.24408903
84	641379283.183897	4802000.9947541	33252902.8998889	206228.642424948	1754065.90365518
85	608376470.788386	4297084.18072456	28450901.9051348	190142.303356551	1547837.26123023
86	573143564.235618	3819082.21185496	24153817.7244102	174423.875493355	1357694.95787368
87	535821028.476159	3368289.99745394	20334735.5125553	159093.529894165	1183271.08238032
88	496633221.737526	2945232.77377373	16966445.5151013	144181.974208419	1024177.55248616
89	455913767.254045	2550707.7296408	14021212.7413276	129734.202766904	879995.578277741
90	414162552.190221	2185963.7432863	11470505.0116868	115819.916623284	750261.375510837
91	372018199.204449	1852381.55160092	9284541.26840052	102526.38015455	634441.458887553
92	330205213.704865	1551116.15764975	7432159.7167996	89947.8928427697	531915.078733002
93	289474730.799583	1282818.42520737	5881043.55914985	78175.7301447883	441967.185890232
94	250544432.15327	1047450.28912729	4598225.13394248	67289.1432733603	363791.455745444
95	214042864.377292	844196.513164743	3550774.84481518	57347.814501677	296502.312472084
96	180471097.271172	671497.798925383	2706578.33165044	48387.7273823528	239154.497970406

97	150180647.834081	527163.147977309	2035080.53272505	40419.6949746537	190766.770588054
98	123363940.814884	408520.10828777	1507917.38474774	33429.4745140302	150347.0756134
99	99973644.1806189	312323.643090543	1099397.27645997	27361.6955186595	116917.60109937
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101	62660218.5563641	174220.60342254	551816.438501375	17680.0774840823	67416.109875341
102	48212965.264916	126463.613202285	377595.835078834	13900.5613892283	49736.0323912587
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104	26426850.2104582	61692.9326640892	161510.680443066	8089.23814436726	25109.5736248017
105	18611928.4930717	40989.7738769787	99817.7477789775	5932.01224469938	17020.3354804345
106	12552331.2620114	26079.7049596297	58827.9739019988	4202.23604573471	11088.3232357351
107	8027328.81303766	15734.1566405922	32748.2689423691	2852.03721291149	6886.08719000041
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111	578882.408437499	898.750836643516	1314.83821068238	301.065722139942	485.54965553847
112	213234.229559995	312.32015512436	416.087374038871	128.008796643507	184.483933398527
113	61979.0876323673	85.6411329516575	103.76721891451	44.0539164093434	56.4751367550197
114	12629.2927495845	16.4630535265664	18.1260859628533	10.9438369915496	12.4212203456762
115	1352.30677974726	1.66303243628693	1.66303243628693	1.47738335412659	1.47738335412659