

# Light Travel

## THIS ISSUE'S PUZZLE

**A GIGANTIC STRUCTURE** with the shape of a 1-degree angle as shown in the picture is placed on the ground. A photon is sent into the structure parallel to the ground, 2617 meters above it. The structure has a mirror-like surface so that each time the photon hits the structure it bounces off with the degree of incidence equal to the degree of reflection. Assuming that the experiment takes place in an empty space, please answer the following questions:

1. How many times will the photon hit the surface of the structure before it flies off for good?
2. How long will the photon be inside the structure (between the first and the last hit)?



## PREVIOUS ISSUE PUZZLES

### Cruel Teacher, Smart Student

Dr. I.M. Meene, a high school physics teacher, is also an inventor. Most of his inventions are very cruel with respect to his students. His latest gadget was designed for lazy students who neglect doing their homework. It has three buttons—red, white, and blue—and a screen on which one can write with a stylus. The teacher pre-selects the number of times each button should be pressed and gives the device to his student. If the student presses the blue button, he needs to write the sentence “I need to do my homework diligently” on the screen as many times as he needs to press the white button. If he presses the red button, the student must write the sentence three times the number of remaining blue button pushes. Finally, if the student presses the white button, he must write the sentence two times the number of remaining red button pushes.

Bobby has not done his latest homework and is up for punishment. Dr. Meene gave him the gadget and told him to push the blue button  $A=5$  times, the red button  $B=2$  times, and the white button  $C=6$  times. Bobby may be lazy, but he’s certainly very astute. He figured out how to accomplish the daunting task with the lightest effort. Assuming the device can’t be tempered with and that Bobby didn’t cheat, please answer the following questions:

1. How many sentences did Bobby write?
2. In how many ways can the smallest number of sentences be written?

Generalization: What are the answers to questions 1 and 2 in terms of  $A$ ,  $B$ , and  $C$ ?

### Solution

1. Bobby wrote 24 sentences.
2. There is only one way.
3. In general, a student could write as few as  $\min(AC, 2BC, 3AB)$  sentences. If  $B = A/2 > C/3$ , there are  $C+1$  ways; if  $B = C/3 > A/2$ , there are  $A+1$  ways; if  $A/2 = C/3 > B$ , there are  $B+1$  ways. If  $B = A/2 = C/3$ , there are  $A+B+C$  ways. Otherwise, there is only one way.

This is a puzzle from a past regional Math Olympiad. The solution presented may not be very elegant but it certainly works.

If  $A = 0$ , then it’s obvious that the student would first press the red button  $B$  times, the white button  $C$  times, and write no sentences. Similar logic applies in case of  $B = 0$  or  $C = 0$ .

If one pushes the red button followed by the white button, it’s advantageous to

the reversed order. Similarly, a blue button push should precede the red button push and the white button push should precede the blue button push. Therefore, the optimal strategy involves continuous sequences of  $(BRW)$ ,  $W(BRW)$  or  $RW(BRW)$ , where  $R$ ,  $W$ , and  $B$  represent one or more successive pushes of the red, white, and blue buttons, respectively, and  $(x)$  indicates repetition of sequence  $x$ .

Consider sequence  $b_1-r-w-b_2$ , where lower case denotes the actual number of pushes. If  $b_1$  were fewer by one and  $b_2$  were larger by one, then the total number of sentences written would be smaller by  $(w-3r)$ , not a function of the number of blue button pushes. This means that unless  $w = 3r$ , sequence  $BRW$  or  $RWB$  is more optimal. If  $w = 3r$ , then  $BRW$  and  $RWB$  are as optimal as  $BRWB$ . What it also means is that you cannot have more than four terms in a sequence as it would imply that one of the sequences  $(b_1+b_2)-r-w-\dots$  or  $r-w-(b_1+b_2)-\dots$  would involve a red button push after a white button push which is not optimal.

Similar logic applies to the other two possibilities, so if a strategy is  $BRW$ ,  $WBR$ , or  $RWB$ , it’s optimal. The number of sentences corresponding to these strategies are  $AC$ ,  $2BC$ , and  $3AB$ , respectively. Hence, the answer. When  $A = 5$ ,  $B = 2$ ,  $C = 6$ , the answer is  $\min(30, 24, 30) = 24$  sentences.

Sequence  $BRWB$  is only optimal if  $C = 3B$ . Analogously,  $RWBR$  is only optimal if  $2C = 3A$  and  $WBRW$  can only be optimal if  $A = 2B$ .

If none of the four-term strategies is optimal, then  $B < A/2 < C/3 < B$  and the solution is unique.

If  $BRW$ ,  $RWB$ , and  $WBR$  are all optimal (and so are all of the four-term strategies), then  $A = B/2 = C/3$ . There are  $A$  ways to achieve the smallest number of sentences for  $BRW$  and  $BRWB$  (start with  $x$  blue button pushes, then all red, all white and  $A-x$  blue,  $x$  could be  $1, 2, \dots, A$ , so there are  $A$  possibilities),  $B$  ways for  $RWB$  and  $RWBR$  and  $C$  ways for  $WBR$  and  $WBRW$ . The total is then  $A+B+C$ .

THIS ISSUE'S CHESS PUZZLE								
White to Move and Mate in Three.								
8								♔
7		♖						
6								
5				♙				♜
4				♚	♙	♖		
3			♗		♙			
2			♙		♚			
1						♙	♚	
	A	B	C	D	E	F	G	H

If BRWB is optimal, then so are BRW and RWB, and  $A/2 < B = C/3$ . There are  $A+1$  possibilities (start with  $x$  blue button pushes, then all red, all white, and  $A-x$  blue,  $x$  could be  $0, 1, \dots, A$ , so there are  $A+1$  possibilities). Analogous reasoning produces the other two answers.

Solutions may be e-mailed to [cont\\_puzzles@yahoo.com](mailto:cont_puzzles@yahoo.com) or mailed to **Puzzles, 25 Sparrow Walk, Newtown, Pa. 18940.**

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by **Nov. 30, 2006.** Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

In our case,  $B \leftrightarrow A/2 \leftrightarrow C/3 \leftrightarrow B$ , so there is only one way to accomplish the task.

**Chess Puzzle**

White to move and mate in three.

Initial position:

White – Ke1, Qg3, Rb5, Bh5, pawns b6, c2, f5, g4.

Black – Kg5, pawns c3, e3, f6, h6.

**Solution**

**Case A**

1. Rb3 e2
2. Qxc3 Kh4
3. Qxf6#

**Case B**

1. Rb3 e2
2. Qxc3 Kf4
3. Qe3#

**SOLVER LISTS**

Due to an administrative deadline, names of only those people who submitted correct solutions by September 30, 2006, are shown on the lists.

**Punishment Puzzle:** *Bob Bartholomew, Bob Byrne, Bill Carroll, William Cross, Andrew Dean, Mark Evans, Gary Faber, Michael Failor, Flick Fornia, Nick Franceschine, Rui Guo, Yehuda Haber, John Hubenschmidt, Jack Krull, Chi Kwok, Raja Malkani, Lee Michelson, Stephen Peeples, David Promislow, Philip Silverman, Sally Smith, Al Spooner, Elnatan Sulimanoff, Kevin Trapp, Virginia Young*

**Chess Puzzle:** *Bill Carroll, John Coffin, Jeff Dvinoff, Raja Malkani, John Marshall, John McCarthy, June Meimban, David Parsons, Mark Peterson, Harry Ploss, Edward Scher, Don Sondergeld, Tim Swankey, Lee Zinzow*

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