

Tax Season Hiring

THIS ISSUE'S PUZZLE

Let's finish the year with a relatively simple math puzzle. It was offered a long time ago at a math competition.

It's November, and an online tax preparation firm, FileWithUs.com, starts to recruit CPAs for the next year as the tax filing season approaches rapidly. The company isn't large, and Mr. Rouge, the manager, plans to finish the hiring process by the end of November.

Mr. Rouge is known to be a very peculiar man, and so is his hiring pattern. During the first day, he plans to fill one position and then one-seventeenth of the remaining positions. (Mr. Rouge knows in advance the number of openings.)

During the second day, he will hire two more people and then fill one-seventeenth of the remaining positions. The pattern continues, and during the final day of hiring, he will employ n people and then fill one-seventeenth of the remaining positions. Please answer the following questions:

1. How many business days will it take Mr. Rouge to complete hiring?
2. How many people will Mr. Rouge hire?
3. How peculiar is the hiring pattern?
4. Is there more than one answer to the puzzle?

PREVIOUS ISSUE'S PUZZLE

ANSWERS TO BLUELAND SOCCER TOURNAMENT PUZZLE

1. $p = \frac{-5 + \sqrt{105}}{10} \approx 0.525$
2. $p = \frac{-2 + \sqrt{34}}{5} \approx 0.766$
3. No such q exists.
4. Chances for the Police in question 1 and for the Army in question 2 are the same (50 percent) regardless of the probability p . Police wins the tournament with 25 percent probability regardless of the probability q in question 3.

The Solution

Miners and Bankers are able to create some inequity in the chances to win the tournament solely due to the asymmetrical scoring system; a combination of a win and a loss is better than two ties. Why in this situation these two teams would tie is unknown. Perhaps in the real world it would never happen. However, it makes an interesting math puzzle. The actual solution could be quite long, so I will try to squeeze the main points within the limits of the puzzles page.

1. There are 243 different scoring combinations. They can be split based on the number of ties (excluding the tie between the Miners and the Bankers) as follows: If the number of ties is t , the number of possible combinations is $2^{5-t} \binom{5}{t}$. If $t = 0, 1, 4$ or 5 , inequities are not explored and the probability of making



ISTOCK

the final match for any team is 50 percent. However, if $t = 2$ or 3 , then the Police team gets an advantage in just a few cases and the expectation of making finals increases to $42 \frac{2}{3}$ (compared to 40) out of 80 ($t=2$) and 22 (compared to 20) out of 40 ($t=3$). The cases could be counted with some work and are left to the readers as an exercise. (Hint: Police will be at an advantage, for example, if one of the cheating teams ties all games.) Total probability of the Police team making the final game is

$$f_t(p) = \frac{1}{2} + \frac{8}{3}p^2 \left(\frac{1-p}{2} \right)^3 + 2p^3 \left(\frac{1-p}{2} \right)^2$$

By setting $f_t'(p)$ to zero, you can find the answer.

2. This is very similar to the first case. Inequities arise when $t = 3$ or 4 , where t is the total number of ties. Expectation of making finals decreases to $18 \frac{2}{3}$ (compared to 20) out of 40 ($t=3$) and 4 (compared to 5) out of 10 ($t=4$). Again, the cases are left as an exercise. (Hint: Army will be at a disadvantage if it ties all three games; this will be offset against the cases in which Police ties all three games.) Total probability of the Police team making the final game is

$$f_w(p) = \frac{1}{2} - p^4 \left(\frac{1-p}{2} \right)^1 - \frac{4}{3}p^3 \left(\frac{1-p}{2} \right)^2$$

By setting $f_w'(p)$ to zero, you can find the answer.

3. Police wins the championship with 50 percent probability if it advances to the final game with certainty. The probability of the Police team's making the finals equals $f(q,p) = qf_t(p) + (1-q)f_w(p)$. Since p is assumed to be uniformly distributed on $[0, 1]$, the following must take place:

$$1 = \int_0^1 f(q,p) dp.$$

By solving the integral equation, you find $q > 1$, which is impossible. A more interesting question: What probability q triggers the expectation of the Police advancing to the final game to be equal to one-half? The same logic applies as we need to solve the equation.

$$0.5 = \int_0^1 f(q,p) dp. \text{ This equation solves for } q = 8/13.$$

4. The old scoring system was fair in that it prevents inequities from occurring. Therefore, the probability of Army's or Police's winning the group stage is 50 percent, regardless of the Miners-Bankers strategy.

Chess Puzzle. White to move and mate in three.
Initial position: White—Ka7, Rf1, Ba2, Bc5, pawns b4, c6, e2. Black—Ke4, pawns c7, d7, e5.

CHESSE PUZZLE SOLUTION.

Case A: 1. Bg8! dxc; 2. Rf7 Kd5; 3. Rf4#
Case B: 1. Bg8! d6; 2. Bg1 d5; 3. Bh7#

SOLVER LISTS

Due to an administrative deadline, names of only those people who submitted correct solutions by Sept. 30, 2005, are shown on the lists.

Soccer Puzzle: Bob Byrne, Mark Evans, Chi Kwok, Lee Michelson, Brian Miller, Al Spooner, Jeanette Woodhall

Chess Puzzle: Mike Crooks, Lee Michelson

THIS ISSUE'S CHESS PUZZLE

White to Move and Mate in Three.
I selected the easiest chess puzzle so far, since the holiday season starts soon and readers may not want to spend a lot of time solving puzzles.

8								
7								
6								
5	♙							
4	♖							
3	♚	♗						
2								
1		♔		♕				
	A	B	C	D	E	F	G	H

Solutions may be e-mailed to cont_puzzles@yahoo.com or mailed to **Puzzles, 25 Sparrow Walk, Newtown, Pa. 18940.**

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by **Nov. 30, 2005.**

Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

ADVERTISER INDEX

To add your company's name to this list, call Mohanna & Associates at 800-800-0341 or e-mail info@mohanna.com.

For links to these advertisers' e-mail addresses and websites, visit the *Contingencies* website at www.contingencies.org/linksto_advert.html.

- Actuarial Careers, Inc.® 5**
914-285-5100
www.actuarialcareers.com
- Actuarial Resources Corp. 49**
913-451-0044
www.arcval.com
- AdminServer. 11**
972-715-2028
www.adminserver.com
- Andover Research Ltd. 6-7**
212-986-8484
www.andoverresearch.com
- Aon 17**
860-773-6253
www.aon.com/ics/cont
- Benfield. C4**
678-297-2629
www.benfieldgroup.com
- Canada Life Reinsurance 23**
416-597-1440
www.canadalife.com/reinsurance
- Chicago Consulting Actuaries. 14**
312-454-3222
www.chicagoconsultingactuaries.com
- COSS Development Corp. 65**
800-776-7087
www.cossdev.com
- CPS Inc. 54**
708-531-8289
mo'connell@cps4jobs.com
- DFA Capital Management 30**
914-701-7200
www.dfa.com
- D.W. Simpson & Company. C2**
800-837-8338
www.dwsimpson.com
- EMB Consulting 15**
858-793-1425
www.embamerica.com
- EWI 43**
972-866-6815
www.ewireinsurance.com
- Hause Actuarial Solutions, Inc. 51**
913-685-2200
www.hauseactuarial.com
- Insureware 36-37,39**
+61 3 9533 6333
www.insureware.com
386-673-1919
- Integrated Actuarial Services. 28**
386-673-1919
- Jacobson Associates. 18**
312-726-1578
www.jacobson-associates.com
- Lynchval 27**
703-709-1000
www.lychval.com
- Mid America Search. 47**
800-200-1986
www.midamericasearch.com
- Milliman 1,3,13,59**
206-624-7940
www.milliman.com
- Palisade. 55**
800-432-RISK
www.palisade.com/contingencies
- PolySystems, Inc. 19,C3**
312-332-5670
www.polysystems.com
- Pryor Associates/ Pauline Reimer 21**
516-935-0100
www.ppryor.com
- Quasar Systems 12**
414-228-8622
www.qsi-r2.com
- Scottish Re (U.S.), Inc. 31**
704-542-9192
www.scottishre.com
- Stewart Search. 69**
888-JOB-OPEN
www.stewartsearch.com
- TAG 63**
www.perotsystems.com
- Towers Perrin 8-9**
904-391-1900
www.towersperrin.com/tillinghast
- Transamerica Reinsurance. 33**
www.transamericareinsurance.com
704-344-2700
- Valani Consulting. 41**
www.valaniconsulting.com/
(416) 417-0181
- Wachovia 50**
800-506-9076
www.wystar.com