Hats Enough!

WARM-UPS

A few hat goodies. The first two are warm-up famous hat puzzles:

► **HAT WARMER NO. 1:** Three blindfolded logicians were told they would have hats placed on their heads—the hats being taken from a sack containing three white and three black hats. Following removal of the blindfolds, each logician would then be able to see the other two along with their hats, but not his own hat. The logicians were told that anybody seeing a white hat must raise his right hand. Anyone able to determine the color of his own hat, was directed to promptly notify the others—that person to be declared the winner.

As it turns out, white hats were placed on each of their heads. When the blindfolds were removed, all three simultaneously raised their hands. After about 10 minutes of silence, one thoughtful logician correctly announced the color of his hat. How did he do it? [Note: The answer to this may be found on the next page. Please do not send in solution.]

► **HAT WARMER NO. 2:** The three logical bears—Papa Bear, Mama Bear, and their daughter, Baby Bee Bear, otherwise known as P, M, and BB—are all seated in a row facing forward, one behind the other. P is last and can see M and BB; directly in front of P is M, who can only see BB; and BB is in front of all and unable to see either P or M.

The bears are told that a hat taken from a pile of hats containing precisely three red and two green hats will be placed on each of their heads (without them seeing which hat was selected). After the three hats have been placed on their heads, P says, " Mmm, I don't know what color hat I'm wearing!" Then M says, " Mmm, and I don't know what color hat *I'm* wearing."

BB says, " Mmm, but I *do* know what color hat *I'm* wearing," and

indeed she did. What color hat was BB wearing and how did she figure it out? [Note: The answer to this may be found on the next page. Please do not send in solution]

THE MAIN EVENT: DRIVE YOU HATTY!

Three brilliant, infallible logicians—A, B, and C—having solved the previous problems, decide to try their luck at another challenging game. Each is blindfolded and randomly draws a hat from a sack that they are told contains five black and three white hats. Each takes two hats and places them on his head, one on top of the other. Their blindfolds are then removed; they face each other in a circle and each can see the hats the others are wearing, but not his own hats.

It's decided that they will take turns trying to deduce which color hats they're wearing—the winner being the first to determine with certainty which hats he's wearing. They decide to go in continual order; first A will try to determine which two hats he is wearing. If he succeeds, he will announce his findings to the group, but if he is unable to determine which hats he is wearing, then it will be B's turn to try. Similarly, if B is unable to determine his hat colors, it will be C's turn; and if at this point C cannot determine his hat colors, they will continue again in order A, B, C, A, B … etc., stopping after a logician has determined which two hats he's wearing.

As it turns out, each of the logicians selected at the start of the game both a black and a white hat.

Which of the three infallible logicians is first to deduce which hats he is wearing, and in which round does he do it?

Please submit your solutions via e-mail to PuzZzles@aol.com [that's with 3 z's!] or by mail to PUZZLES, 17 Ravine Rd., Great Neck, NY 11023. Please submit answers as soon as possible to make the solvers list. Solvers list will be limited to the first 100 solvers and only the first correct attempt will count toward the solvers list. And please, send any ideas or any favorites for consideration for future issues to the same addresses.

Incidentally, as I have decided to limit my tenure as puzzle editor to two years, this is my last issue. I appreciate having had this forum. The readers and solvers were wonderful. I'll really miss the column and "solvers" who always amazed me with their innovative approaches, persistence, thoughtful appreciation, and humor. Still, as they say in actuarialese—hats enough!

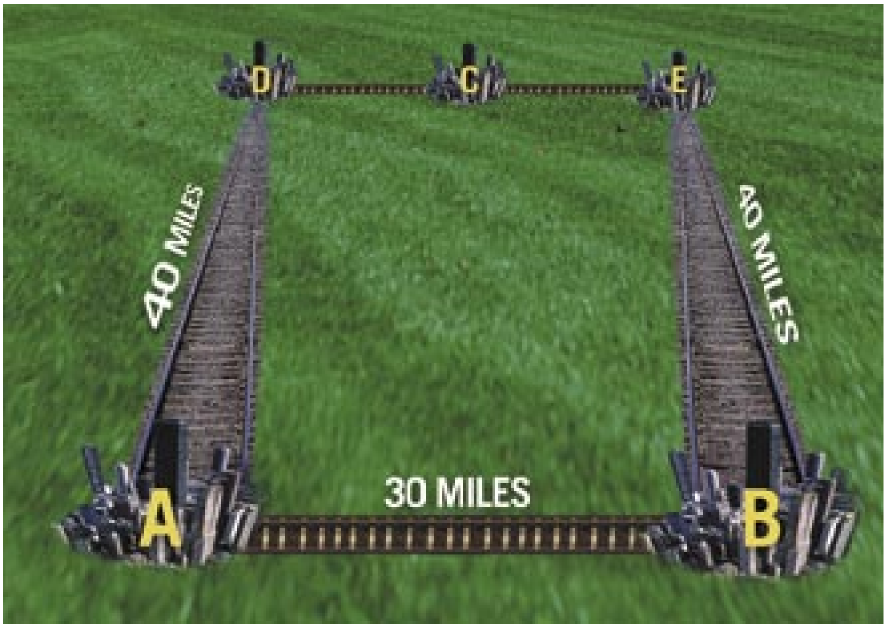
Last Issues Puzzle

Consider three cities: A, B, and C. B is 30 miles east of A. C is 40 miles north of the precise center between A and B. (Disregard the earth's curvature; assume we're talking about a flat world with parallel and Euclideanly straight longitude and latitudes.)

1. It's desirable to be able to connect each city by rail, but we want to minimize the amount of track required. Can you design the shortest railroad track that would enable connection between any two cities? What is the amount of track required for such?

2. Now suppose there's also a city D that's 40 miles north of A, and a city E that's 40 miles north of B. Can you design the shortest railroad track that would enable connection between any of the four cities, situated in a rectangle, of A,B,D, and E? Is there more than one solution? What's the amount of track required for such?

Very difficult: Can you design the shortest track that would be required to enable connection between any of the three cities A, B, and D? What is the amount of track required for such?



Answers (all unique)

PUZZLE NO. 1. 40+15*√3 or approximately 65.98 miles—this is made by a Y-shaped track with an interior point, P, 5*√3 north of the center of the base. One approach is to solve using some basic calculus. As it turns out, angle APB is 120 degrees.

PUZZLE NO. 2. 40+30*√3 or approximately 91.96 miles—this is made by a stick figure with two interior points: 5*√3 north of the center of AB 5*√3 south of the middle of BD. Interior obtuse angles again 120 degrees.

VERY DIFFICULT. Most solvers solved this with assorted numerical approximation techniques. The minimal track distance is approximately 67.66 miles [or more precisely 10*sqrt(25+12*√3)]. As it turns out, the optimal solution for any given triangle ABC (having no angle greater or equal to 120 degrees) is a Y pattern with interior point P such that all angles—APB, BPC, and APC—are equal to 120 degrees. Several readers sent in rigorous proofs of this fact.

This interior point, which can then be easily determined using trigonometry, is known as the Fermat point or as the Torricelli point. It can be obtained by constructing equilateral triangles on each side of the given triangle, and then connecting the farthest vertex of each constructed triangle with the opposite vertex of the original—the intersection of these three lines yielding the required point. The interested reader might consider the minimal travel problem for objects with more than three points or beyond two-dimensional space.

Solvers

B. Bartholomew, P. Bottelli Jr., M. Crooks, A. Dean, R. Dowsett, D. Engelmayer, M. Evans, M. Failor, Y. Fridman, T. Friesen, S. Gallancy, B. Goldberg, B. Kester, C. Kwok, K. Larsen, D. Liewellyn, R. Link, L. Michelson, J. Moran, P. Morse, A. Narale, D. Onnen, S. Peeples, A. Schallhorn, B. Shroyer, A. Spooner, R. Stokes, T. Swankey, D. Thaller, A. Torelli, Z. Wadia, D. Wille, T. Wille

Additional Jul/Aug solvers

J. Herder, F. Walton

Answers to Hat Warmers

▶ **HAT WARMER NO. 1:** At first, no one could figure out which hat he was wearing. After a while, however, the most astute logician of the three reasoned as follows: If he had been wearing a black hat, then both the others would have deduced that they were each raising hands on account of each other's white hat. Thus, they each would have hurriedly blurted out that they were each wearing white hats. Since the other two did not hurriedly blurt anything, he realized he did not have a black hat.

▶ **HAT WARMER NO. 2:** P doesn't know his own hat color, so obviously he didn't see two green hats in front of him. If BB had had a green hat, then M would have reasoned that since P didn't know which hat he was wearing, her own hat must be red (i.e. she would have announced her hat as being red). Therefore, since M didn't announce her red hat color, BB realized that M must be looking at her red hat!