Loot Distribution

THIS ISSUE'S PUZZLE

Some of you may remember a computer game puzzle I offered last year. That puzzle was given at an actual job interview. Here is another problem that a job applicant had to solve to get a job.

Eight bandits robbed a bank for $10,000, which they took in $100 bills. The robbers are ranked according to their prior conviction record with the most senior thief being the one with most felonies committed. In order to split the 100 bills among themselves, the following procedure was established.

The most senior felon proposes a distribution of the loot. All of the robbers then vote and if 50% or more agree, the proposed distribution takes place. Otherwise, the proposal is rejected and the most senior bandit gets killed. Then the second most senior robber proposes a distribution and the process continues until a plan is voted for.

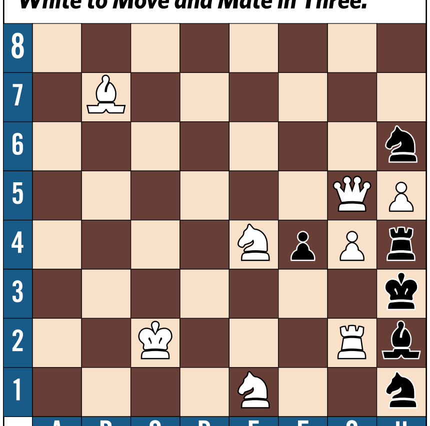
The robbers are also perfect logicians (can this happen in real life?) so they know how each of them will vote. There are no grudges to be held during votes as the thieves do not believe in emotions. Being rational, they have preferences: first is to remain alive, next is to end up with as much money as possible and finally, if given a choice between otherwise equal outcomes, to have fewer thieves to divide the loot. The most senior bandit proposes a plan that maximizes his money and which he knows won't get rejected.

The moment he puts his proposal up for a vote the ninth thief shows up. This is the one who was on the lookout and the one that all other robbers "forgot" about. He was ranked last. Now that he is in the picture, he has to be taken into account.

Will the proposal change? If so, how different are the proposals? How much money will the latecomer get? Show all work.

THIS ISSUE'S CHESS PUZZLE

White to Move and Mate in Three.



PREVIOUS ISSUE PUZZLES

Remarkable parabola

A mathematician tired after a day full of productive research at his university decided to relax for a while doing …some more math work. He drew a parabola with 0 and 1 as x-intercepts and a maximum of 1 at x = ½. He then picked a number between 0 and 1 on the x-axis and looked to see what the corresponding y-coordinate is. That y-coordinate became his new x-coordinate and the mathematician looked at the new y-coordinate. The process continued and to his surprise, the fourth x-coordinate (which was non-zero) did not change from the third. Obviously, the process went into an infinite loop after that.

What was his starting x-coordinate? (Note that a closed-form solution is needed.)

The mathematician was so excited that he soon developed a generic formula for the starting position if x-coordinates start to repeat after n moves. Can you do the same?

Solution

1. $x_1 = \sin^2(1/12*\pi)$ or $\sin^2(5/12*\pi)$.
2. $x_1 = \sin^2((\pi k + \pi/2) / (3*2^{n-2}))$, k is an integer and k–1 is not divisible by 3.

(Alternatively, $x_1 = \frac{2 \pm\sqrt{2 \pm \ldots\sqrt{2 \pm\sqrt{3}}}}{4}$, where the number of ± is n–2.)

I've seen a few various solutions that lead to results in at least two different formats—algebraic and trigonometric. I like the latter better and will show it here. In addition, the trigonometric solution is truly closed-form because it does not contain iterative features. Only four readers found the trigonometric solution—Leonid Shteyman, Lee Michelson, David Promislow and Virginia Young. The solution shown is generic, so the partial case is just a by-product of the answer.

From the y-intercept and the maximum given you should determine the parabola equation as $y = -4x^2+4x$, or $y = 4x(1-x)$.

Because x_1, the starting position, is located between 0 and 1 we can always have a number z such that $x_1 = \sin^2 z$.

The second x-coordinate, $x_2 = y(x_1)$, as mentioned in the problem, is then equal to $4\sin^2 z(1-\sin^2 z) = \sin^2(2z)$. Analogously, one can show that $x_n = \sin^2(2^{n-1}z)$.

If x-coordinates start to repeat after n moves, we have $x_{n+1} = x_n$, or $\sin^2(2^{n-1}z) = \sin^2(2^n z)$.

The latter equation is a combination of the two equations, $\sin(2^{n-1}z) = \sin(2^n z)$ and $\sin(2^{n-1}z) = -\sin(2^n z)$.

There are four solutions to these equations (k is an integer):

1. $x_1 = \sin^2(\pi k / (3*2^{n-2}))$.
2. $x_1 = \sin^2(\pi k / 2^{n-2})$.
3. $x_1 = \sin^2((\pi k + \pi/2) / (3*2^{n-2}))$.
4. $x_1 = \sin^2((\pi k + \pi/2) / 2^{n-2})$.

However, some of these solutions result in an $x_n = 0$, but the problem asks for a non-zero solution. Solutions 2 and 4 are thus eliminated.

We know that the repetition can only be at ¾, so we should make sure that while $\sin^2(2^{n-1}z) = ¾$, $\sin^2(2^{n-2}z)$ should not be ¾. Therefore, solution 1 is eliminated because the repetition will start earlier.

In addition, there is a restriction on the integer k. In solution 3, the remainder of the division of k by 3 cannot equal to 1.

Finally, if n = 3, we get our partial case.

From solution 3 we have $x_1 = \sin^2((\pi k + \pi/2) / 6)$, k–1 is not divisible by 3, so that x_1 could either be $\sin^2(5/12*\pi)$ or $\sin^2(1/12*\pi)$.

Chess Puzzle

White to move and mate in three.

Initial position: White—Kh7, Rb6, Rh6, Nd5, Ng8, Bg6, pawns b3, c3, c4, g7. Black—Kc5, Bb1, pawn b7.

Solution—Case A

1. Kh8! (zugzwang) Bc2
2. Bd3 any black move
3. Rb5#

Solution—Case B

1. Kh8! (zugzwang) Bd3
2. Be4 Bxc4
3. b4#

Solution—Case C

1. Kh8! (zugzwang) Bxg6+
2. Rbxg6 b6
3. Rc6#

SOLVER LISTS

Due to an administrative deadline, names of only those people whose correct solutions were received by the puzzles department editor on or before March 31, 2007, are shown on the lists.

Parabola Puzzle: *Bob Bartholomew, Bob Byrne, Bill Carroll, Mark Danburg-Wyld, Mark Evans, Mike Failor, Rui Guo, John Hubenshmidt, Janusz Kawczak, Philip Lehpamer, Tim Luker, Jerry Miccolis, Lee Michelson, David Oakden, Don Onnen, Stephen Peeples, Harry Ploss, David Promislow, Noam Segal, Levi Self, Leonid Shteyman, Philip Silverman, Don Sondergeld, Al Spooner, Tony Torelli, Kevin Trapp, Virginia Young*

Chess Puzzle: *Bill Carroll, Leigh Halliwell, Janusz Kawczak, Todd Kennedy, Robert Koch, Krishna Kothoor, Philip Lehpamer, Brian Liebeskind, Tim Luker, John McCarthy, June Meimban, Mark Mercier, Lee Michelson, Don Onnen, Harry Ploss, Don Sondergeld, Andrew Witte, Lee Zinzow*

Solutions may be e-mailed to cont_puzzles @yahoo.com or mailed to Puzzles, 25 Sparrow Walk, Newtown, Pa. 18940.

In order to make the solver lists (separately maintained for the regular and chess puzzles), please make sure that your answers and solutions are received by May 31, 2007. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.