Logicians at King Louis's Dinner

First, a belated welcome to Yan Fridman, who last issue succeeded Noam Segal as the creator of our puzzles page. Yan comes to us originally from the Ukraine, where he was raised until his family emigrated in 1996. He graduated from the National Technical University of Ukraine with a master's in applied mathematics in 1996.

"I didn't learn about the actuarial profession until I got to the States," he says, "where my sister's friend, who worked in Coopers & Lybrand in San Francisco as a computer programmer, told me about it. She gave me the phone number of the Society of Actuaries and the address for its website. I started taking exams in 1997 and became an FSA in November 2003."

He worked at GE Financial Assurance and ING as a life insurance actuary and at Towers Perrin as a pension actuary. He's currently with the American Life Insurance Co. in Wilmington, Del.

"I always liked puzzles," he says, "mostly math and logical puzzles. In Ukraine, math Olympiads were popular, and Olympiad problems were very much like puzzles. I also like whodunits and some chess puzzles. I'm always looking for puzzles, be it in *Contingencies*, on math websites, or other media."

Last October, Yan and Noam met for the first time in front of the *Contingencies* booth at the SOA annual meeting in New York. The combined brainpower was positively sizzling.

This Issue's Puzzles

Noam Segal in the Nov.–Dec. 2004 issue of *Contingencies* offered three puzzles related to logicians and their hats. Here is a follow-up puzzle.

Three brilliant, flawless logicians—A, B, and C—having solved all hat problems so far, found themselves at a dinner party hosted by a tyrant king Louis the Horrible. King Louis was known to be very generous to anyone who could solve his puzzles. At the same time, he did not tolerate if someone was not up to the challenge, and the result was beheading.

Each of the three logicians was blindfolded and randomly drew a hat from a sack that contained three hats with positive integers written on each hat. Each logician took a hat and placed it on his/her head. Their blindfolds were then removed; they faced each other in a circle and each could see the hats the others were wearing, but not his/her own hat.

They knew that two of the numbers added up to the third. In order to be generously rewarded they needed to figure out what number was written on their hats.

Here is the conversation that took place:

Logician A: "I don't know what my number is."

Logician B: "I don't know what my number is."

Logician C: "I don't know what my number is."

Logician A: "Now I know what my number is. It is 50."

Immediately afterwards the other two logicians knew their numbers.

What are the other numbers, and why did Logician A speak only at Round 2?

Solutions to Last Issue's Puzzles

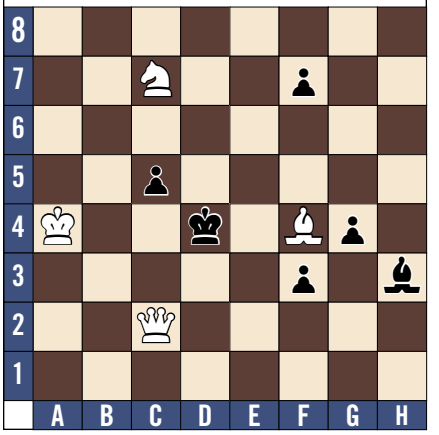
Puzzle 1. Mr. White needed to make sure that he would be able to determine which bottle is bad just by looking at the weight shown by the pharmacy scale. It means that there should be a unique way to find the defective bottle. There are surely many ways to do so, but the most optimal way is to take n pills from the nth bottle (numbered 0 through 9). There will be 0+1+…+9 = 45 pills put on the scale. If pills in all bottles weighed 100mg, the total weight would then be 45*100mg = 4500 mg. If bottle n contained 90mg pills, the total weight would be 4500mg – n*10mg, and this is precisely how Mr. White would do his job. The exact formula is as follows: bad bottle # = (4500mg – total weight) / 10mg.

Another answer, which would require more pills, is to take n pills from the nth bottle (numbered 1 through 10). I accepted both answers. Incidentally, this puzzle is very popular. A variant of it appeared in the movie *The Columbo: Bye-Bye Sky-High I.Q. Murder Case*. The puzzle was solved by Mrs. Columbo.

Puzzle 2. Mr. White used a similar idea to weigh the pills. In this case by subtracting the total weight from the "ideal" weight ("ideal" corresponds to having no bad bottles), he would not only determine the number of defective bottles but also identify them. It is clear the Puzzle 1 approach would not work. The optimal approach in this case is the following: Take 2^n pills from the nth bottle (numbered 0 through 9). The difference between the ideal weight and the actual weight is divided by 10. It is then written in the binary notation with leading zeroes, if any, so that 10 positions are shown. Bad bottles are represented by digit "1" by reading from right to left. For example, notation 0001000100 means that bottles 2 and 6 are defective.

CHESS PUZZLE

White to Move and Mate in Three



Puzzle 3. This puzzle is a natural extension of the previous case. We allowed the weight of pills to vary. There could be 10 different weights. The optimal way is to do pretty much the same as in Puzzle 2 but write one-tenth of the difference between the ideal weight and the total weight in our usual, decimal notation. In order for a number in that notation to be unique, each position needs to be uniquely determined. It is done by taking 10^n pills from the nth bottle (numbered 0 through 9). One-tenth of the weight of the pills in a bottle is determined by the difference between 10 and the corresponding digit by reading from right to left. For example, notation 000075209 means that pills in Bottle 0 weigh (10-9)*10mg = 10mg, pills in Bottle 4 weigh (10-7)*10mg = 30mg, and pills in Bottles 5 through 9 weigh 100mg.

Solutions may be e-mailed to
cont_puzzles @yahoo.com
or mailed to
Puzzles, 25 Sparrow Walk,
Newtown, Pa. 18940.

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by March 31, 2005.
Depending on the response volume, solver lists may contain only the names of people who solved puzzles on first attempt.

Chess Puzzle

Initial position.
White: Kc1, Ba2, Rf2
Black: Ka1, c3

Solution:

1. Bg8 – c2
2. Rf7 – Ka2
3. Ra7x

All black moves are forced.

I'd like to note here that the first person to solve it was Kevin Trapp's 13-year-old son, Jordan.

Solvers

Due to an administrative deadline and overwhelming response, names of only those people who submitted correct solutions on first attempt by Jan. 31, 2005, are shown on the lists.

Pharmacy Puzzle: *Bob Bartholomew, Francis Bernardi, Bob Byrne, Lois Cappellano, Mark Danburg-Wyld, Francis de Regnaucourt, Brad Dow, Lance Dyrland, Robert Ellerbruch, Mark Evans, Michael Failor, Jerry Francis, John Herder, Anna Hoover, Scott Humpert, Vladimir Itkin, Dennis Jacobs, Frank Karlinski, Richard Kollmar, Kevin Larsen, Robert Link, Steve Mathys, Stefano Merlo, Lee Michelson, David Oakden, John Pauly, Stephen Peeples, Bob Powell, Steve Ruiter, Noam Segal, Michael Sipos, Don Sondergeld, Al Spooner, Elnatan Sulimanoff, Tony Torelli*

Chess Puzzle: *Richard Botelli, Bob Byrne, Robert Ellerbruch, Len Helfgott, Clive Keatinge, Michael Lamb, Steve Mathys, June Meimban, Lee Michelson, Don Onnen, John Pauly, Igor Pogrebinsky, David Promislow, Edward Scher, Noam Segal, Al Spooner, Ronald Stokes, Jordan Trapp*