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Breaking Into the Black-Scholes Black Box

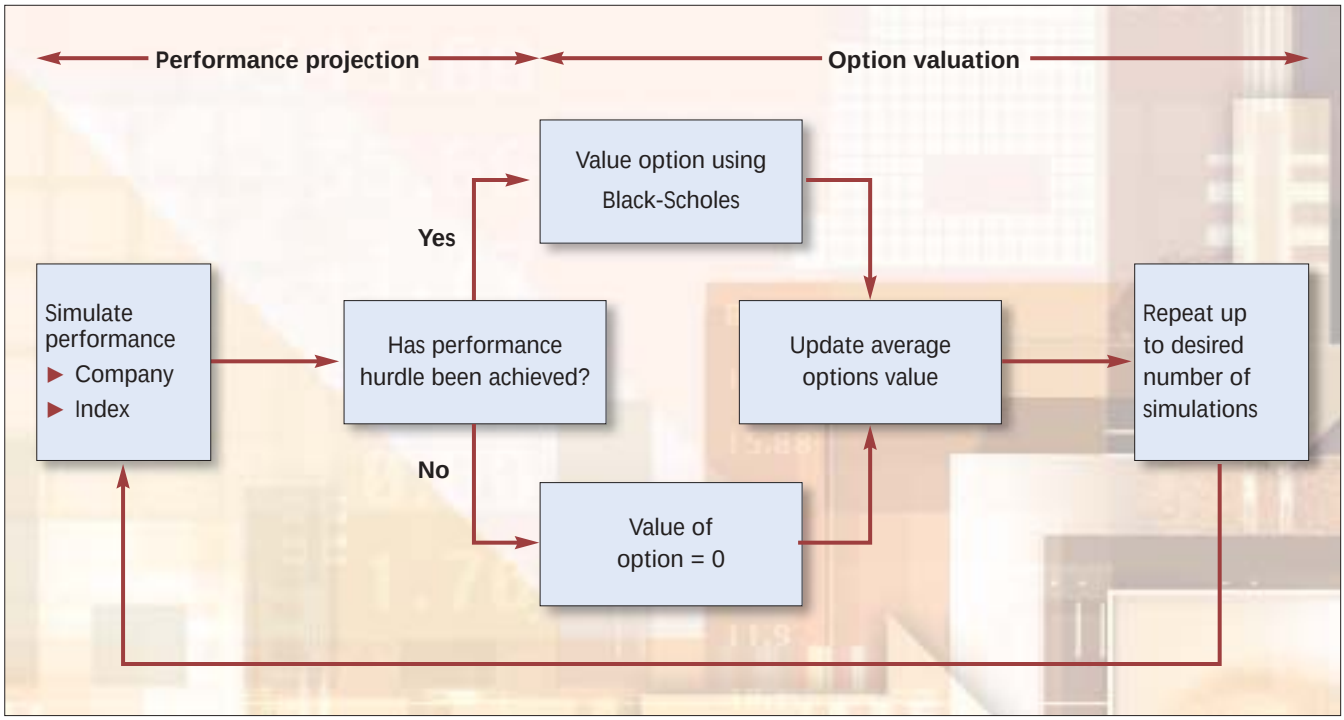
Actuaries and Executive Option Valuation

New proposed accounting rules will mean that executives can no longer be so secretive about their stock options packages. Actuaries will be able to play a major role in making them more transparent.

Executive share options (ESOs) can be an efficient means of aligning the interests of shareholders and management. Shareholder discontent with the lack of coherent disclosure of these arrangements, and in some cases the level of executive reward, has created pressures for greater transparency. The obvious path looms large: If executives want to keep their options packages, better disclosure and accounting principles will need to be applied.

The response of the International Accounting Standards Board (IASB) to this issue is Exposure Draft ED 2. If adopted, this accounting standard will require companies to expense ESOs in their profit-and-loss statements. Many will view this as a positive step. A number of issues, however, require serious further consideration. Moving the cost of ESOs from the footnotes to the sharp end of the accounts will increase the focus on the valuation techniques employed. Such techniques will need to be rigorous and defensible, or the benefits of better disclosure will be lost.

Therein lies an opportunity for the actuarial profession. Currently, most ESOs are disclosed only in the footnotes of company accounts and valued



using the Black-Scholes formula with various ad hoc adjustments to allow for early exercise, forfeiture, and the like. Many of these adjustments are technically unsound. In addition, to make shareholders more comfortable with the award of ESOs, many companies are incorporating additional performance hurdles into ESO structures. These hurdles add an extra dimension and complexity to the valuation process. Actuaries can help to develop and apply valuation techniques that address these issues in a both justifiable and rigorous manner.

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Plain Vanilla and Beyond

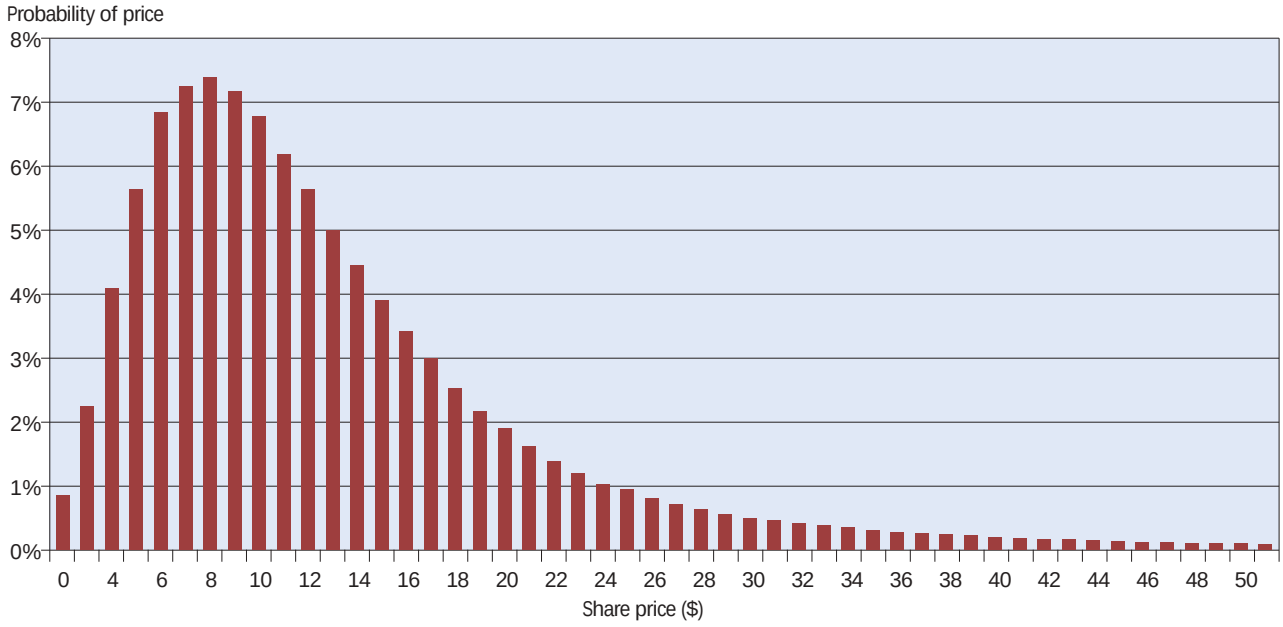
Vanilla ESOs are traditional share (call) options. The executive can typically exercise the option at a future time providing that the company's share price is above the ESO's strike price. For companies with such plans, valuation should be relatively straightforward. Given the widespread usage of the Black-Scholes valuation formula and the absence of a commonly accepted alternative, it would be difficult to promote a valuation methodology that diverged materially from the Black-Scholes approach.

Many options packages, however, have more complex benefit designs—a complexity that often arises from the inclusion of performance hurdles. That is, even if the share price is above the option's strike price, the executive cannot exercise the option until certain performance criteria are met. Examples of performance hurdles include:

- The share price must be greater than some fixed level, say 20 percent greater than the grant date share price.
- The company's cumulative total shareholder return must outperform that for a given index.
- The company's cumulative total shareholder return may be compared to those for a peer group of companies. The number of options vesting in the executive will be a function of the percentile ranking of the company's total shareholder return.

There can be further complications. The testing of the above

Company share price distribution



Future share price band $	Probability of share price being at least $X	Probability of being in band	Share price $	Payoff $	Nominal value (cents)	present value (cents)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0–10	41.6%	41.6%	—	—	—	—
10–15	69.6%	28.0%	12.29	2.29	64.25	48.81
15–20	84.8%	15.2%	17.23	7.23	109.82	83.41
20–25	92.3%	7.5%	22.22	12.22	91.94	69.83
25–30	96.0%	3.7%	27.22	17.22	63.76	48.43
30–35	97.9%	1.9%	32.23	22.23	41.27	31.35
35–40	98.8%	1.0%	37.24	27.24	26.07	19.80
40–45	99.3%	0.5%	42.25	32.25	16.38	12.44
45–50	99.6%	0.3%	47.27	37.27	10.33	7.85
50+	100.0%	0.4%	58.74	48.74	18.67	14.18
		100.0%				336.10

criteria for vesting purposes can occur at a fixed point, such as three years from the grant date. Thereafter, the vested option might be exercisable up to the expiry date of five years from the initial grant date. Alternatively, there could be a window of opportunity for vesting, say from three years post grant date until the expiry of the option. Under this scenario, the performance criteria may need to be met only once for the option to vest in the executive.

Such options can't be accurately valued using the standard Black-Scholes formula. To value ESOs with complex performance hurdles and to explore the impact of early exercise, the paths taken to get to the final price distribution are required. Fortunately, such adjustments can be made in a way that's also consistent with the Black-Scholes framework. Achieving this consistency requires the adoption of a risk-neutral environment and the lognormal distribution of future share prices inherent in Black-Scholes.

The time has come to crack open the Black-Scholes black box and peer inside.

This task was taken on by two Australian actuaries (Bernard

Wong and Paul Carrett) whose response was to employ a combination of risk-neutral valuation theory and simulation techniques. This approach is documented in their paper, "Executive Options: Valuation and Projection Methodologies," published by the Institute of Actuaries of Australia, and available on www.actuaries.asn.au.

The diagram below, taken from the paper, outlines their approach. In short, one projects the risk-neutral returns of the executive's company and, where applicable, those of the peer group or index. For a given simulation, these returns are projected until the performance hurdle is achieved and the option vests in the executive. At this point the option transforms into a vanilla ESO that can be approximately valued using the Black-Scholes formula. This value must then be discounted back to the grant date to give the value of the option for a single simulation. For simulations where the performance hurdle is not hit, and hence the option does not vest, a zero value is recorded. This process is repeated many thousands of times. The average option value obtained across all simulations provides an estimate of the executive option's value, allowing for the impact of the performance hurdle.

This abbreviated description glosses over some difficult modelling problems, problems familiar to those who have worked in the area of stochastic modelling. It was necessary to address such issues as random number generation, Cholesky decomposition, and variance reduction techniques. Interested readers are referred to the paper's appendices for a discussion of these topics.

The Perils of Ad Hoc Adjustments

Rather than indulge in the complexities of simulation modelling, can't "rule of thumb" adjustments be used to produce similar results? The short answer is "generally no." In fact, many of these ad hoc adjustments are outright misleading, to the extent that their use could seriously undermine the usefulness of the proposed accounting standard. The following example illustrates this point.

Consider the following simple performance hurdle:

Hurdle:	If at the end of the term the total shareholder return (TSR) of the company's stock ranks in the top 50 TSRs when compared against a comparator group of 100 nominated companies, then the ESO vests in the executive.

The executive's company ESO is to be valued using the information in the table below:

Issue Price	$10.00
Strike Price	$10.00
Volatility	25% pa
Term	5 years
Risk-Free Rate	5.5% pa

For simplicity, we assume that dividends are not payable on either the company's shares or the comparator group shares.

At first glance, we might expect this hurdle to reduce the value of the ESO to approximately 50 percent of the raw Black-Scholes value, based on the expectation of a 50:50 chance of the company's shares ranking in the top 50 out of 100 shares. However, this would be incorrect as ESOs already have a built-in hurdle that produces a zero payoff if the eventual underlying share price doesn't exceed the exercise price. The interaction between the strike price hurdle and the performance hurdle matters a great deal.

As discussed, to incorporate a TSR comparison into the valuation we would need to project the TSRs for each company in the comparator group over the relevant term, and then perform the ranking comparison. This can't be done accurately using the Black-Scholes formula. However, we can see the impact of the TSR hurdle by considering the following:

- All the share prices of the companies in the comparator group would be projected on the same assumptions with the exception of volatility as per the Black-Scholes risk-neutral framework.
- In the projection, volatility affects the spread of potential share prices around an average share price value, not the average value itself. Therefore, all the companies in the comparator

group would have the same expected share price increase.

Based on our example, we expect the average share price of each company will increase by around 32 percent (i.e., 5.5 percent per year compounded continuously for five years) at the end of the five-year term. The expected increase consists of some potential increases that are above and some that are below this average.

In addition, we would also expect the 100 shares as a group to have an average share price increase of approximately 32 percent. This would again consist of some shares increasing by more than 32 percent and some increasing by less. If a share's price increased by significantly more than the average increase, it's reasonable to assume that it has outperformed 50 percent of the peer group and would therefore vest.

The share price distribution and value for the ESO of our example can be summarised as follows:

(Column (1) is the price band within which the projected future share price of the share underlying the ESO can fall. (2) is the probability that the share price will be less than the upper bound of the price band. (3) is the probability that the share price will be in the given band. (4) is the expected share price within the band. (5) is the payoff, the expected share price less the Exercise Price of $10. (6) is the probability of that payoff being achieved, i.e., (3) x (5). (7) is the nominal value from (6) discounted at the risk-free rate of 5.5 percent for five years. The sum of column (7) an estimate of the Black-Scholes value of the ESO.)

As detailed above, we're expecting all share prices to increase by 31.7 percent on average. This return would give an average share price of $13.17 at the end of the five-year projection period for the share underlying our ESO. We would expect the ESO to vest if the return is significantly higher than the average return. Taking a 50 percent increase compared to 31.7 percent ($15 as opposed to $13.17) as significant out performance, we can see the impact of the hurdle by eliminating any value attributable to potential share prices below $15. In this case, there's a 41.6 percent chance the potential share price will be less than $10 and a 28.0 percent chance of it being between $10 and $15. The potential share prices below $10 contribute no value to the ESO because they're below the exercise price, while the potential share prices between $10 and $15 contribute 48.81 cents of the total ESO value of 336.10 cents.

That is, a 50 percent TSR hurdle will reduce the value of this ESO by only 14.5 percent, even though there's a 69.6 percent probability that our ESO will not vest.

The need for great care when adjusting for performance hurdles, and the need for understanding the impact of benefit design on value, is obvious. It's to be hoped that either professional practice and/or accounting standards will address this issue to ensure that ad hoc adjustments don't lead to misleading valuations or a lack of comparability between companies.

Other Features of ESOs

The IASB draft standard refers to two further issues for which adjustments should be made—non-marketability and early exercise.

Non-marketability. It's argued that because an executive can't sell the ESO (due to there being no apparent market) its value is reduced relative to a traded option. That is, a traded option can be sold for a price that captures both the intrinsic and time value of the option. On early exercise of non-marketable ESOs, only the intrinsic value is obtained. Further, the executive is likely to have restrictions placed on his ability to hedge this exposure by trading in his company's shares.

An approximate procedure to allow for non-marketability might be to use the full term of the ESO for valuation purposes, but with an adjustment for early exercise (as per below). The impact of non-marketability will be reflected in the early exercise of ESOs.

Early exercise. The Black-Scholes formula assumes that the ESO will be exercised at the end of the term. Most ESOs allow exercise before this time. At first glance, it would appear that this could be allowed for by using Black-Scholes with a modified term. For example, if ESOs issued in the past with a contractual term of five years actually had an average life of four years, then the term in the formula could simply be changed to four years. Indeed, this is what the IASB draft suggests.

However, we would argue that this is incorrect. In general, ESOs are only voluntarily exercised early if there is a significant payoff. Hence, a better approximation would be to model the incidence of early exercise based on instances where the potential share price exceeds a given value or increase. This can be modelled using Monte Carlo techniques.

What's It All Mean?

In short, improved disclosure of ESOs is coming. There's a real risk, though, that the effectiveness of the new IASB disclosure regime could be undermined by misleading adjustments or inconsistent practices between companies. The actuarial profession needs to become involved now to influence outcomes in this important aspect of corporate governance.

In Australia, the profession has started down this path. A committee has been set up, and the circulation of a draft valuation guidance note is in the offing. We would strongly urge our North American colleagues to consider taking similar action.

Globally, the profession has an opportunity to undertake high-profile work in the area of executive options. The interest of the accounting profession, regulatory bodies, and executives is not likely to diminish. We should take advantage of this opportunity to apply our mathematical and financial skills to this challenging and important area. ●

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