More Logicians

THIS ISSUE'S PUZZLE

You may recall that late in 2004 and in the beginning of 2005 we published a few logician puzzles. Here's another one of the kind, suggested by Bob Gabriel and revised to shorten the computation time.

Two brilliant, perfect logicians, Green and Blue, are given the following information regarding the two positive integers A and B: They are both greater than 1, A < B, and their sum is less than 40.

Green is told the product of A and B, and Blue is given the sum of A and B. The logicians are then engaged in the following conversation:

Green: "I know the product."
Blue: "I know the sum."
Green: "I don't know the sum."
Blue: "I already knew that."
Green: "I now know the sum."
Blue: "I now know the product."

Please answer the following questions:
1. What are the numbers A and B?
2. Is there only one solution?
Show all work.

PREVIOUS ISSUE PUZZLES

Fight With the Dragon

A scary and always hungry dragon has appeared in a peaceful village and has a habit of eating at least five villagers every day. Many citizens have tried to fight the dragon off, but to no avail; They've inevitably become meals for the voracious monster.

Finally, a brave soldier, Cornelius, gets his lucky sword, goes to the center square, which the dragon had occupied, and challenges the three-headed monster to a duel. With every swing of his sword, Cornelius can miss or chop one or two of the dragon's heads. If Cornelius chops off one head, a new head immediately pops up in its place (even if the dragon has only a single head). If he removes two heads with one stroke, the monster loses them, and they don't grow back. If the soldier misses, the dragon grows a new head.

The probability of Cornelius missing with the swing of the sword is the same regardless of the number of heads. If the monster has two or more heads, the soldier is as likely to take two heads off as to miss altogether. The dragon gets more strength every time Cornelius misses. In fact, the moment it gets five heads, it will eat the soldier alive. On the other hand, once the dragon loses all of its heads for good, it's killed. The fight must continue until the deadly end.

Please answer the following questions:

3. What are the soldier's chances of winning?

4. (Requires computer calculations) How many times need Cornelius be as likely to chop off two heads as to miss in order to make it a fair duel (fair means the winning chances are equal for both fighters)?

Chess Puzzle

White to move and mate in three.

Initial position:
White - Kb6, Bg2, pawns a7, h3.
Black - Ka8, pawns b7, e4, f5, h4.

SOLUTIONS FOR THE MAY-JUNE PUZZLES

Fight With the Dragon

Answers:

1. 60%

2. a ≈ .802, the only solution of the equation $a^3 + 2a^2 - a - 1 = 0$ on [0, 1].

SOLUTION:

I hope that readers will not chastise me for not being mathematically strict in stating the following fact: If the dragon has *n* heads, then the probability of the soldier's victory is the same regardless of how long the battle has been going on. I am not going to prove this statement here, but eager readers can consider this as an exercise.

If you believe me, then the solution becomes straightforward. If not, there are many other ways to solve it, including building a matrix of possible states and finding the limit of the *m*th power of the matrix with *m* approaching infinity to find the answer.

Let x_n be the probability that the *dragon wins* if the monster has *n* heads. Then the following system of equations will produce the answer. Each equation is based on possible state changes from time *t* to time *t* + 1, where states represent the number of the dragon's heads.

For example, if the dragon has four heads, then after the next sword swing, it can have five heads following a failure to chop head(s) off (with probability *q*), four heads (with probability $1 - p - q$), or two heads (with probability *p*), hence the fifth equation.

$$\begin{cases} x_0 = 0 \\ x_1 = q \cdot x_2 + (1-q) \cdot x_1 \\ x_2 = q \cdot x_3 + (1-p-q) \cdot x_2 + p \cdot x_0 \\ x_3 = q \cdot x_4 + (1-p-q) \cdot x_3 + p \cdot x_1 \\ x_4 = q \cdot x_5 + (1-p-q) \cdot x_4 + p \cdot x_2 \\ x_5 = 1 \end{cases}$$

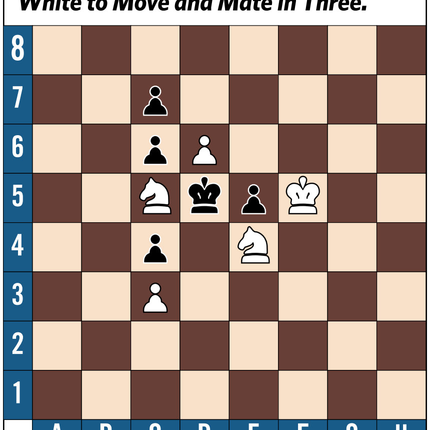
This system effectively represents four equations with four unknowns. By solving the system and denoting $a = p/q$ and $y(a) = a^3 + 2a^2 + a + 1$, we get:

$$\begin{cases} x_0 = 0 \\ x_1 = 1/y(a) \\ x_2 = 1/y(a) \\ x_3 = (a+1)/y(a) \\ x_4 = (a^3 + 2a^2 + 2a + 1)/((a+1) \cdot y(a)) \\ x_5 = 1 \end{cases}$$

To answer question 1, we set $a = 1$, $n = 3$ and get $x_3 = 2/5$, so the probability of the soldier's victory is $3/5$, or 60%.

THIS ISSUE'S CHESS PUZZLE

White to Move and Mate in Three.



To answer question 2, we need x_3 = .5, so from this we get a = .802 (requires computer).

Chess Puzzle

1. Bh1 f4
2. Bxe4 f3
3. Bxb7#

SOLVER LISTS

Due to an administrative deadline, names of only those people who submitted correct solutions by May 31, 2006, are shown on the lists.

Dragon Puzzle: *John Adduci, Steve Altschuld, Bob Bartholomew, Richard Bottelli, Scott Boulay, Geoff Bridges, Bob Byrne, William Carroll, John Cook, Mike Crooks, Tom Dahl, Mark Danburg-Wyld, Andrew Dean, Robert Ellerbruch, Mark Evans, Mike Failor, Steve Gallancy, Nick Gifford, William Glasgow, Mark Glickman, Brian Goldberg, Yehuda Haber, John Hubenschmidt, Stuart Klugman, Chi Kwok, Kevin Larsen, Tim Luker, Howard Mahler, Raja Malkani, Ken McKusick, Lee Michelson, Carl Nauman, Don Onnen, Stephen Peeples, Luke Porter, David Promislow, Justin Radick, Francis de Regnaucourt, Philip Silverman, Todd Simkin, Greg Scruton, Al Spooner, Ron Stokes, Elnatan Sulimanoff, Doug Szper, Donn Takebayashi, Tony Torelli, Kevin Trapp, David Uhland, Jenny Young, Darin Zimmerman*

Chess Puzzle: *Casey Abell, Steve Altschuld, Ian Arvin, Steven Azar, Richard Bottelli, Scott Boulay, Geoff Bridges, Brian Brugger, Robert Burrell, Bob Byrne, William Carroll, Alan Clark, Mike Crooks, Rob Drozd, Jeffrey Dvinoff, Andy Dvorine, Brian Eastman, Robert Ellerbruch, Mark Evans, Mike Failor, Chuck Fuhrer, Bob Gardner, Colby Gardner, William Glasgow, Mark Glickman, Andrew Gordon, Leigh Halliwell, Len Helfgott, Norm Henricks, Dave Heyman, Bob Howard, Will Joseph, Greg Keklak, Todd Kennedy, Robert Koch, Ignace Kuchazik, Kevin Larsen, Brian Liebeskind, Dave Llewellyn, Tim Luker, Don Mac Donald, Raja Malkani, John Marshall, Ken McKusick, June Meimban, Mark Mercier, Randy Meyer, Lee Michelson, Jim Murray, William Nibbelin, Don Onnen, Wade Oshiro, David Parsons, Mark Peterson, Matthew Petro, Igor Pogrebinsky, Dylan Porter, David Promislow, Justin Radick, Hugh Ramler, Boris Raskin, Francis de Regnaucourt, Robert Rietz, Dave Scherr, Greg Scruton, Noam Segal, Don Sondergeld, Al Spooner, Ron Stokes, Thomas Struppeck, Thomas Tholany, Kevin Trapp, Howard Wachspress, Zorast Wadia, Gerald Ward, Andrew Yerre, Hank Youngerman, Darin Zimmerman, Lee Zinzow*

Solutions may be e-mailed to cont_puzzles @yahoo.com or mailed to Puzzles, 25 Sparrow Walk, Newtown, Pa. 18940.

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by July 31, 2006. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

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