Beating the House

THIS ISSUE'S PUZZLE

I was running up against my deadline without a good idea for a puzzle, so I decided to pay another visit to my eccentric friend Maxwell Chance and see what he was up to. Having a slightly off-kilter buddy you can call upon is practically indispensable if you want to write puzzles.

Max, I regret to report, was in sad shape. He had recently been discharged from his employer when it was discovered he was spending most of his working hours on an Internet gambling site. Naturally, I scolded him about such behavior, but having access to scenarios involving gambling and/or bad employee behavior is also practically indispensable if you want to write puzzles.

Max explained that he had cracked the code of a particular online casino's roulette game. You may be familiar with roulette (although I hope not from personal experience). For the sake of this puzzle, the key information is that you can bet on the wheel coming up "red" or "black" with a "double-or-nothing" payout, although a fair bet would also include "green." At any rate, Max explained, he had discovered that for this particular online casino, if you recorded the color outcome of 16,342 consecutive spins and fed it into a computer program he had written, it would tell you the outcome of the next 10 spins with 90 percent accuracy. That is, the algorithm would tell you, in order, what the next 10 spins would be, but exactly one would be wrong, although you couldn't tell which one until it happened.

This seemed to Max to be easy money, so he was spending a lot of time online doing just that. Unfortunately, he told me, it wasn't paying off.

"It's the one error that's killing me. I want to maximize my return, so I bet it all each round and when I hit the error, I go bust. Then I have to go pawn something and start the whole process over again. It takes me hours to get the algorithm set again, and by the time I'm ready, I'm unable to think of a smarter betting strategy," Max explained.

"Tell you what," I said. "You start the algorithm over, I'll lend you $100, and I'll come up with a strategy to maximize the outcome without going bust." Now this was unscrupulous behavior—we should have been reporting the data flaw to the casino operator instead of profiting from it. But having loose moral standards is occasionally indispensable if you want to write puzzles.

Two questions:

1. If we have to set the amounts for each of the 10 bets in advance, what is the best we can do?

2. If we can set each bet after seeing the outcome of the prior bet (thus knowing when the one wrong result comes through), what is the best we can do?

Please spell out the strategy, not just the result. And, please hurry! Max has up to 16,311 consecutive spins entered in his algorithm.

PREVIOUS ISSUE'S PUZZLE

Hurrah for Our Pirate King!

This puzzle runs in November, so to make it topical, it involves elections. And to make it tropical, it involves pirates. What could be better than a puzzle involving pirate elections?

So the seven pirate kings from the seven seas gathered for their convention (held every seven years) to discuss major important pirate issues of the day. They drank grog, sang some Gilbert & Sullivan, and then settled in to elect the king of the pirate kings.

In the first round of the election, every pirate voted for himself to be the king of the pirate kings. This was not unexpected. Captain Ahab suggested that they hold a second vote, where they each voted for their second choice. This also resulted in a seven-way tie. Captain Bluebeard suggested they each vote for their third choice, which also resulted in a seven-way tie. Captain Cook suggested they each vote for their fourth choice, which also resulted in a seven-way tie. Then Captain Drake had an idea: that they each vote for their fifth choice, which also resulted in a seven-way tie. Captain Edwards suggested they each vote for their sixth choice, which also resulted in a seven-way tie. Captain Fear had the bright idea that they each vote for their seventh choice, but it was pointed out that they had already voted six times and so everyone already knew who everyone's seventh choice was, and it was also a seven-way tie.

Things looked pretty bleak. But then Captain Gary spoke up, and said, "There are seven of us, so no two-way vote can end in a tie. Why don't we first vote between Captains Ahab and Bluebeard, then pit that winner against Captain Cook, and so forth, until we've all had a go at it?"

Captain Drake grumbled that such an arrangement gave an unfair advantage to the later rounds, but these were pirates and not mathematicians, so they proceeded according to Captain Gary's plan.

Captain Drake was quite correct, and Captain Gary (who was a mathematician as well as a pirate) knew it. So two questions for you:

If Gary can choose the order of the voting, can he always rig the election so that he is certain to win?

If the pirates just go alphabetically, what is the probability of Gary winning?

You may take as given that every pirate votes for his top preference in each round and doesn't change his preference from round to round.

Solution

1. If Gary can choose the order of the voting, can he always rig the election so that he is certain to win?

Yes, by the pigeonhole principle. It's possible to arrange the preferences so that a pirate can win as many as five head-to-head votes, but each pirate must have at least one head-to-head vote that he can win. Working backward from (one of) the pirates he can beat, Gary can always rig the election so that anyone who would beat him is eliminated before he enters the vote.

2. If they just go alphabetically, what is the probability of Gary winning?

This portion was tougher than I'd intended. Most people who wrote in either assumed that the pirates each had three head-to-head wins or were told they could take that as a simplifying assumption if they asked, so I will take that case first.

In the first round, Ahab and Bluebeard each have a 50 percent chance of winning. Say (without loss of generality), Ahab wins. In the second round, Ahab has only a 40 percent chance of winning—he beats just two of the remaining five pirates. That means Cook has a 60 percent chance of winning Round 2. Say that Ahab wins again. In the third round, he has only a 25 percent chance of winning. That puts Ahab with a 5 percent (=50 percent * 40 percent *25 percent) chance of being the winner after Round 3. Cook has a 24 percent chance of being the winner after Round 3 (60 percent from Round 2 * 40 percent chance of winning his second vote). If Ahab wins the third round, he has no chance of winning the fourth round. The odds of winning for the new candidate are 66 percent (= 100 percent – 24 percent – 5 percent *2) in Round 3, 67.6 percent in Round 4, 66.36 percent in Round 5, and 66.7 percent in Round 6—this final figure being Gary's probability of winning.

The complication, of course, is that there aren't exactly three head-to-head wins for each pirate. The set of all possible preferences is the same as the set of all unique Latin squares of order 7, of which there are 61,479,419,904,000. However, given that each pirate votes for himself first, you can divide that by 7!— but I resorted to simulation at that point. Based on around 2 million scenarios, approximately 68 percent of scenarios offer three head-to-head wins per pirate, 31.8 percent of scenarios have at least one pirate who wins four head-to-head votes, and the remainder have at least one pirate who wins five head-to-head votes. This improves the odds for pirates who come earlier in the process, slightly. I got Gary's odds of winning at 66.4 percent.

Solutions may be e-mailed to cont.puzzles@gmail.com or mailed to Puzzles, 65 W. 35th Place, Eugene, Ore. 97405.

In order to make the solver list, please make sure that your answers and solutions are received by Jan 31, 2009. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.

SOLVERS LIST

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