Cable Company Mathematicians

THIS ISSUE'S PUZZLES

This puzzle, although in a different form, was suggested by Neil Neubarth. I have slightly revised the problem to make it more difficult.

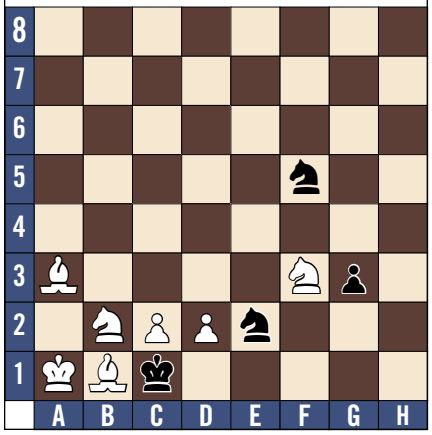
A small cable company, which operates in a rural area, is struggling for cash. Its management tried many ways to get revenue, but nothing seemed to help. When it was clear that the company would soon fold, its president came up with a wonderful idea: employ mathematicians who could minimize the cost for expensive TV cable. Fortunately, the new employees quickly solved the problem and reduced cable expenses by minimizing cable length.

All rooms require cable installation, are $b \times 12 \times 12$ feet (length × width × height). The cable connector in a TV set is located on the back of the set on one of the 12-×-12 walls (ignore the gap between the TV and the wall), one foot from the floor (ceiling and floor are considered walls in this problem), and six feet from each of the side walls. The cable outlet is on the opposite 12-×-12 wall, one foot from the ceiling and six feet from each side wall (what a good idea to install the outlet so high!). The cable cannot be connected through the air; it has to go along the walls (alas, it may spoil room design). Cable is thin and can go behind the TV along the wall.

Calculate the minimum cable length required and how the connection should be organized for each of the three room types in the area, whose length b equals: (1) 10 feet; (2) 20 feet; (3) 30 feet.

THIS ISSUE'S CHESS PUZZLE

White to Move and Mate in Three.



SOLUTIONS FOR THE NOV.-DEC. PUZZLES

Tax Season Hiring Answers

(1) 16; (2) 256; (3) The pattern is not peculiar at all. Mr. Rouge hires 16 people every day for 16 days. (4) No. This solution is unique.

Thanks to all for submitting your answers and solutions. Last issue's puzzles generated the largest response in 2005.

I hope you enjoyed puzzles in 2005, and I look forward to working with all of you in 2006. Special thanks to Bob Byrne and Al Spooner, who submitted correct answers to all regular puzzles in 2005!

There are a number of methods of solving the puzzle. Here is one of them.

Let a_t be the number of positions remaining to be filled at the end of the day t, and m is the total number of people to hire. Then

$a_0 = m$

$a_t = {}^{16}\!/_{17}(a_{t-1} - t)$

$a_n = 0$

By recursion one can show that

$0 = a_n = ({}^{16}\!/_{17})^n \cdot m - [({}^{16}\!/_{17})^n \cdot 1 + ({}^{16}\!/_{17})^{n-1} \cdot 2 + ({}^{16}\!/_{17})^{n-2} \cdot 3 + \ldots + ({}^{16}\!/_{17})^1 \cdot n]$

After recognizing that the sum in brackets can be calculated as the sum of n geometric progressions, the equation will reduce to the following:

$16n - 256 = ({}^{16}\!/_{17})^n (m - 256).$

The left side is always integral. Because 16 and 17 are mutually prime numbers, the only way for the right side to be integral is for m, the number of openings, to be equal to 256. This leads to $n = 16$. (I ignore the trivial solution of $m = n = 1$.)

Mr. Rouge will hire one person plus ${}^{1}\!/_{17}(256 - 1) = 15$, a total of *16* people on day 1; two more people plus ${}^{1}\!/_{17}(256 - 16 - 2) = 14$, a total of 16 people, and so on.

The reason I asked question No. 4 is to encourage readers to solve the puzzle as opposed to guessing the answer and proving it will fit the conditions of the problem.

Chess Puzzle

Case A—(1) Bf2! Kb4; (2) Bd4 Ka3; (3) Bc5#. All black moves are forced.

SOLVER LISTS

Due to an administrative deadline, names of only those people who submitted correct solutions by Nov. 30, 2005, are shown on the lists.

Hiring Puzzle: Bob Bartholomew, Bob Byrne, William Carroll, Mike Crooks, Dan Czabaj, Andrew Dean, Darrell Doddridge, Rob Drozd, Brian Eastman, Bernie Erickson, Mark Evans, Mike Failor, Bill Feldman, Brett Hodgson, Frank Knorr, Calvin Lai, Warren Leisinger, Dennis McNeese, Brian Miller, David Oakden, David Promislow, Andy Ribaudo, Noam Segal, Al Spooner, Elnatan Sulimanoff, Tom Toce, Tony Torelli, Kevin Trapp, Virginia Young

Chess Puzzle: Cliff Anderson, Michael Bavalsky, Jan Brown, Robert Burrell, William Carroll, Mike Crooks, Rob Drozd, Brian Eastman, Alan Erlebacher, Mike Failor, Bob Gardner, Leigh Halliwell, Frank Knorr, Tim Luker, Arnie Malaski, Raja Malkani, Jim Marks, John Marshall, Mark Mercier, Carlos Nava, David Oakden, LeRoy Pruitt, Boris Raskin, Noam Segal, Kevin Trapp, Frank Walton, Terry Watson, Paul West, Paul Wiesike, Andrew Yerre, Rich Yurkowitz

Solutions may be e-mailed to cont_puzzles @yahoo.com or mailed to Puzzles, 25 Sparrow Walk, Newtown, Pa. 18940.

In order to make the solver lists (separately maintained for the regular and chess puzzles), please submit your answers and solutions by Jan. 31, 2006. Depending on the response volume, solver lists may contain only the names of people who solved puzzles on the first attempt.