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Strategies for Ranking DFA Results

DYNAMIC FINANCIAL ANALYSIS (DFA) is often used to provide quantitative insights into strategic business questions. These questions include:

- Which reinsurance program is better?
- Which asset mix is preferred?
- What is the impact of changing product mix?
- What are the ramifications of changing pricing strategy?

BY GENERATING HUNDREDS OR THOUSANDS OF SIMULATIONS, DFA allows its users to estimate key measures of reward and risk. Examples of such measures include return on equity, standard deviation of return on equity, and the probability of certain adverse events such as earnings or surplus falling below stated thresholds. By comparing these measures across the different strategies being considered, the user can draw inferences regarding the strategies.

Many discussions of DFA present the concept of an efficient frontier—the set of all points that maximize the reward measure for each level of risk. Fig. 1 shows an illustration of an efficient frontier in which the reward measure is mean return on equity and the risk measure is the standard deviation thereof.

For single-period models with many simplifying assumptions, constrained optimization models can be specified and solved for the asset mix strategies that form the efficient frontier. For other strategic decisions and for multi-period evaluations, such models aren't easily solved. Instead, DFA models can be used to calculate the reward and risk measures for a limited number of alternate strategies. Thus, the corresponding chart to fig. 1 from DFA often contains a limited number of points, such as is shown in fig. 2.

One question for the user is then the choice of strategies from among those tested. This article provides several suggestions for ranking strategies in the context of a case study focusing on the choice of investment strategy. The approaches suggested, though, can be applied to any type of strategic decision, including those that focus on more than one aspect of an insurance company's operations—such as a set of strategies for mitigating risk through changes in both investment strategy and reinsurance program.

FIGURE 1

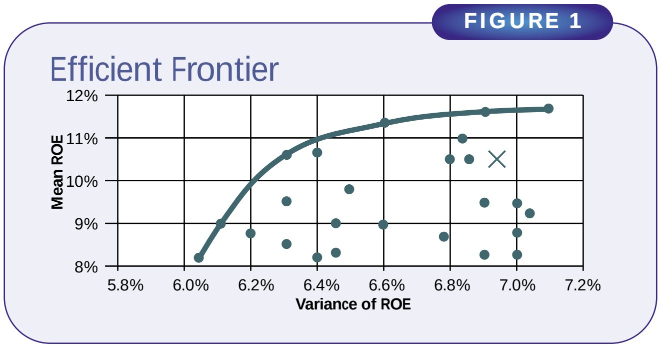
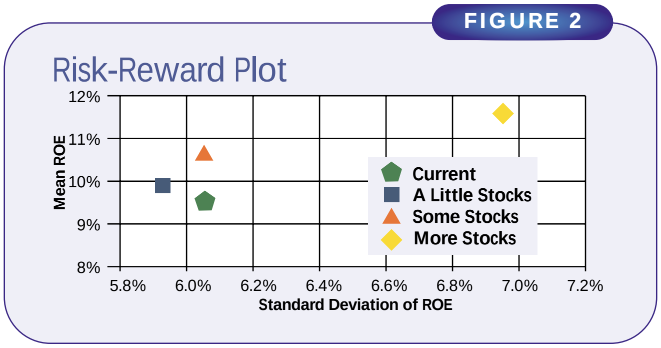


FIGURE 2



Case Study

The company used to illustrate this case study is a multistate workers' compensation insurer. It has grown steadily and profitably over the past several years. It has maintained a conservative investment portfolio and is considering alternatives for enhancing return. We'll assume the company is publicly traded and would like to maintain a path of stable earnings growth. Therefore it has a strong preference for strategies that have consistent profits from year to year. In light of that preference, the company has purchased reinsurance to maintain a relatively low per-claim retention.

Because the focus of this discussion is ranking strategies, rather than the details of the modeling process, the DFA model has been kept quite simple, which may lead to distortions in the modeled values of the correlation between company and market returns.

The following assumptions identify the models of the key underwriting variables:

- Exposure growth is assumed to vary from company expectations due to changes in wage inflation and a random component using a Normal distribution.
- Average rates are assumed to increase with loss cost trends on an expected value basis, subject to deviations from loss cost trends ranging from +5 percent to –5 percent in any calendar year.
- The loss costs per dollar of payroll are assumed to grow by 2 percent per annum with a Normally distributed random component.
- Variable and fixed underwriting expenses are assumed to match company expectations.

The current asset portfolio contains 12.5 percent in govern-

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TABLE 1

Alternate Investment Strategies

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
Government	12.5%	10.0%	7.5%	7.5%
Non-Taxable	55.0%	52.5%	50.0%	35.0%
Corporate	31.5%	31.5%	26.5%	26.5%
Stocks	0.0%	5.0%	15.0%	30.0%
Cash	1.0%	1.0%	1.0%	1.0%
Total	100.0%	100.0%	100.0%	100.0%

FIGURE 3

Risk-Reward Plot

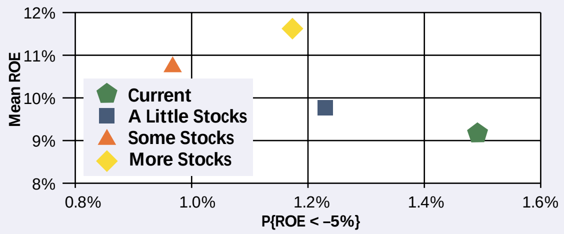

TABLE 2

Risk and Reward Measure

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
Prob. ROE<–5%	1.5%	1.3%	0.9%	1.1%
Beta	9.4%	9.8%	10.7%	11.8%

TABLE 3

CAPM Regression Results

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
Alpha	0.039	0.038	0.036	0.032
Beta	0.086	0.133	0.231	0.378

TABLE 4

Excess Return Sharpe Ratio

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
Mean ROE	9.4%	9.8%	10.7%	11.8%
Std. Dev. of ROE	6.0%	5.9%	6.1%	7.0%
Excess Return, Sharpe Ratio	0.80	0.89	1.01	1.05

ment bonds, 55 percent in municipal bonds, 31.5 percent in corporate bonds, and 1 percent in cash. Bonds are held so that 25 percent (of par value) mature within five years, 55 percent between five and 10 years, 17.5 percent between 10 and 20 years, and 2.5 percent between 20 and 30 years. This mix varies by type of bond (government, municipal, and corporate). No stocks, real estate, or mortgages are currently held. The alternate investment strategies being considered are shown in Table 1.

The model assumes that assets are sold and purchased, as necessary, in the middle of each calendar year to attain the target mix. For simplicity, the analysis ignores transaction costs. The impact of any realized capital gains or losses on sales of assets is, of course, reflected.

The company uses a five-year planning for evaluation of alternate strategies. The company's measures of reward and risk are mean return on economic net worth and the probability that economic net worth decreases one year to the next, respectively. Economic returns (i.e., with all bonds valued at market and reserves evaluated on a risk-adjusted discounted basis) are used because many of the calculations presented later in this article require market returns. Although there are many differences between changes in market value and return on economic equity, the DFA model can't estimate the market value of the company so return on economic equity is used as an approximation.

For the current asset mix, the values of these statistics derived from the DFA model are a 9.4 percent mean return on economic equity (ROE) and a 1.5 percent probability that economic equity decreases by 5 percent or more.

Table 2 and fig. 3 show the values for the selected measures of reward and risk for each strategy. The addition of stocks generally reduces risk and increases reward. The "current" and "a little stocks" strategies would be considered sub-optimal, since they have lower returns than "some stocks" and "more stocks" strategies, but are more likely to lead to reductions in GAAP net worth of at least 5 percent.

If we also consider standard deviation as a risk measure, as shown in fig. 2, the "a little stocks" strategy also appears optimal.

Selection of Preferred Strategy

In light of the findings in Table 2 and Figs. 2 and 3, the stock strategies appear to be preferred over the current strategy, but the preference among them isn't obvious. The company needs to be able to rank these strategies. The first of the six possible approaches suggested in this article uses the capital-asset pricing model (CAPM). The second approach is based on Sharpe's ratio. The third approach uses a concept underlying insurance pricing. The fourth approach uses the Markowitz Mean-Variance model to calculate rankings. The fifth approach uses proportional hazards to rank the strategies. The last approach is a combination of the CAPM and proportional hazards approaches.

Capital-Asset Pricing Model Approach

In CAPM, the ROE required by the market is expressed as a risk premium in excess of the risk-free rate, where the magnitude of the risk premium is calculated as a ratio to the risk premium for the market as a whole. The ratio is referred to as Beta and is a measure of the covariance between the company's ROE and the market ROE. Beta can be estimated using linear regression, as shown in Equation 1 (Harry H. Panjer (ed.), *Financial Economics*, The Actuarial Foundation, Schaumburg, Illinois, 1998, p. 391).

$$R_i - r = \alpha_i + \beta_i(R^M - r) + \varepsilon_i \quad (1)$$

In this formula, r is the return on a risk-free investment, R^M is the ROE for the market, and R_i is the observed ROE for the strategy being tested.

In this application, the company's economic ROE for each year in each iteration is used as the independent variable. Total return on stocks is commonly used as a measure of total market return. Total return on stocks is available from the macroeconomic scenario generator in the DFA model. The short-term government rate is a measure of the risk-free rate and also is available from the macroeconomic scenario generator in the DFA model. Over the 1,000 iterations run for this case study, the 5-year average risk-free rate (r) was 4.6 percent and the 5-year average market return (RM) was 10.8 percent. Using Equation 1, Alpha and Beta for each investment strategy can then be estimated. These values are shown in Table 3.

For strategies with values of Alpha that are positive, the ROE is simulated to be higher than is required to meet investor demands. (It's important to determine whether any excess returns are due to arbitrage opportunities embedded in the DFA model's macroeconomic scenario generator, rather than in the company's investment strategy. In this case, we haven't evaluated the extent to which inconsistencies in the macroeconomic scenario generator have led to the values of Alpha that are greater than zero. The results presented here are intended solely to be illustrative.)

The first approach uses this information to rank the strategies. As can be seen, under this ranking system, the current strategy has the highest excess return as measured by Alpha. If the present value of each year's net income at the required return as determined by CAPM had been calculated, the strategies would be ranked in the same order as they are using Alpha.

Excess Return Sharpe Ratio

The Excess Return Sharpe Ratio (www.stanford.edu/~wfsharpe/mis/rr/miz_rr5.htm) is the ratio of the mean ROE in excess of the risk-free rate as a percentage of the standard deviation of ROE, as shown in Equation 2.

$$Ratio = \frac{E[ROE] - r}{\sigma_{ROE}} \quad (2)$$

The values needed to calculate the Excess Return Sharpe Ratio and the resulting ratios are shown in Table 4.

The "more stocks" strategy ranks highest using this ratio.

Insurance Pricing Approach

In his paper "On the Theory of Increased Limits and Excess of Loss Pricing" (Proceedings of the CAS, Vol. LXIV, 1977, p. 43), Miccolis proposes that the profit (and contingencies) margin used in insurance pricing be a function of the variability of results. He proposes that either standard deviation or variance be used. Specifically, he presents the formula shown here in Equation 3.

$$Premium = E[y] + \lambda * Var[y] \quad (3)$$

If that formula is considered in the context of company returns, it could be restated as Equation 4.

$$ROE = r + \lambda * \sigma^2_{ROE} \quad (4)$$

In this formula, r is the risk-free rate, ROE is the mean ROE, and σ^2_{ROE} is the variance of the ROE for the strategy of interest. In this discussion, variance will be used as the measure of variability. Variance could be replaced with any other measure of variability. The use of variance in these illustrations isn't intended to express a preference for this risk measure over any other. If standard deviation were used and $\lambda = 1$, this measure would be equivalent to the Excess Return Sharpe Ratio.

A high value of lambda indicates that the company is receiving a high return relative to the variability of results, whereas a low value of lambda indicates a low return relative to variability. Estimated values of lambda can therefore be used to rank strategies.

The ROEs, ROE variances, and lambdas calculated for each

TABLE 5

Lambda Results

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
ROE	9.4%	9.8%	10.7%	11.8%
ROE Variance	0.0037	0.0035	0.0037	0.0048
Lambda	13.25	14.96	16.70	15.04

TABLE 6

Markowitz Mean-Variance Scores τ=0.075

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
Score	0.0105	0.0112	0.0124	0.0129

TABLE 7

Markowitz Mean-Variance Scores τ=0.04

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
Score	0.0039	0.0043	0.0049	0.0046

TABLE 8

Proportional Hazards Adjusted Net Income (in $ millions) for Year 1

ρ	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
1.5	4.1	4.3	4.6	4.8
2.0	3.3	3.5	3.8	3.9

TABLE 9

Present Value of Proportional Hazards Adjusted Net Income (in $ millions)

ρ	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
1.5	16.5	16.2	15.3	13.6
2.0	12.0	12.1	11.8	10.5

strategy for the sample company are shown in Table 5.

This approach suggests that the "some stocks" mix provides greater return relative to risk than the alternate approaches under consideration.

Markowitz Mean-Variance Model

The Markowitz mean-variance model provides another framework for ranking strategies. Specifically, according to Panjer, an investor's efficient portfolio is one that meets the criterion shown in Equation 5.

$$\max\{2\tau\mu_x - \sigma_x^2\} \quad \text{subject to} \sum_{i=1}^{N} x_i = 1 \qquad (5)$$

In this formula, τ is a measure of the investor's risk tolerance, μ and σ are the mean and standard deviation of ROE, and x is the investment mix across N asset classes.

In most situations, τ is not known directly. However, it can be approximated by first estimating the τ underlying the current portfolio. The calculated value can then be adjusted based on discussions with company management. According to Panjer, τ can be estimated from the results of a DFA model of strategy x using Equation 6.

$$\tau = \frac{Var(R_x)}{E[R_x] - r} \qquad (6)$$

For the sample company's current investment strategy, τ is estimated to have a value of 0.075. The results of the DFA model for the alternate investment strategies and the selected value of τ can be substituted into Equation 5 to derive scores for each strategy. These scores are shown in Table 6.

Using this ranking approach, the "more stocks" strategy is preferred.

If the company's risk tolerance is lower, say τ = 0.04, the scores and the rankings change, as shown in Table 7.

As expected, if the company has lower risk tolerance, the "some stocks" strategy is preferred to the "more stocks" strategy.

Proportional Hazards

Proportional hazards (Wang, Shaun, "Insurance Pricing and Increased Limits Ratemaking by Proportional Hazards Transforms," *Insurance, Mathematics and Economics* 17, pp. 43-54) is similar to the Markowitz Mean-Variance Analysis in that it explicitly reflects an assumption regarding the company's level of risk tolerance. In the proportional hazards approach, the results from each iteration are ordered from worst to best. $x(i)$ will denote the i^{th} ordered result. The proportional hazards mean is calculated as:

$$\text{Adjusted mean} = \sum_{i=1}^{n} x(i)[F(i)^{(1/\rho)} - F(i-1)^{(1/\rho)}] \qquad (7)$$

Because Monte Carlo simulation was used to derive the results, $F(i) = i/n$. Therefore, (7) is equivalent to:

$$\text{Adjusted mean} = \sum_{i=1}^{n} x(i)[(i/n)^{(1/\rho)} - ((i-1)/n)^{(1/\rho)}] \qquad (8)$$

ρ is the measure of risk tolerance. If ρ =1, the adjusted mean is equal to the unadjusted mean. As ρ increases, more weight is given to the adverse results. As such, the result of the calculation provides a weighted average result that is lower than the arithmetic mean. A company with higher risk aversion would therefore select a higher ρ.

Table 8 shows the weighted average economic net income for the first year using two different values of ρ.

Under both values of ρ, the "more stocks" strategy is preferred. A review of the results of each iteration indicates that the higher standard deviation of results for the "more stocks" strategy emanates from greater upside risk rather than downside risk. In fact, most of the downside risk appears to emanate from underwriting results, not investment results. Therefore, the higher returns on investment attained as more stocks are added to the portfolio offsets the adverse underwriting results in some of the iterations. The operating losses in those iterations are therefore generally smaller as the percentage of stocks is increased. As such, giving more weight to the adverse scenarios increases the preference for the "more stocks" strategy.

Proportional Hazards with CAPM

The proportional hazards approach, as presented above, is applied to each year individually. To incorporate the simulated results for all of the projection years, the proportional hazards adjusted economic income for each year can be discounted using the required ROE from CAPM. That is, for each year, the proportional hazards adjusted mean economic income is calculated. The present value of these adjusted means is then calculated using the required ROE from the CAPM model. The results of this approach are shown in Table 9.

The higher income from the "more stocks" strategy isn't sufficient to offset the higher required return emanating from the greater correlation of company and market results. Using both values of ρ, the "more stocks" strategy is the least preferred. With the lower value of ρ (higher risk tolerance), the current strategy is preferred. As ρ increases, the "a little stocks" strategy becomes preferred because the "penalty" for the adverse underwriting results has a greater impact on the current strategy as the company perceives itself to be more risk averse. This greater reduction of income as the result of the downside variability for the current strategy causes the net income for that strategy to be so low that it more than offsets the higher discount rate required for the "a little stocks" strategy.

Comparison of Methods

Before summarizing the findings of these approaches, it's important to understand the differences among them. The key sources of difference are:

- Whether the approach considers all risk observed by the company or just systematic risk;
- Whether the approach implicitly assumes that the ROEs are normally distributed;
- Whether the approach uses "market" information regarding the company's level of risk aversion or allows the company to select its level of risk aversion.

Types of Risk Considered

Some risk measures, such as the standard deviation of ROE, incorporate all the modeled risks. These risks, though, can be separated into systematic and nonsystematic risk. Systematic risk is the portion of variability that's correlated with the broader market and therefore can't be diversified by investors. Nonsystematic risk is the company-specific portion of risk. Nonsystematic risk can be diversified.

Of the approaches presented here, only CAPM looks solely at systematic risk. Therefore, a company evaluating its risk-reward profile from the perspective of an outside investor would want to consider one of the approaches based on CAPM.

The other approaches (other than the combined proportional hazards/CAPM approach) consider the total risk the company faces. Companies that view risk and reward from the perspective of threats to the company's operations might prefer these approaches.

Why is nonsystematic risk important? DFA, at least at this stage in its development, doesn't usually reflect all a company's frictional costs of having adverse results. If a company loses a significant portion of its surplus, for example, it will need to take corrective action, such as reducing the amount of business written, purchasing more reinsurance, or issuing debt. In addition, its credit rating may suffer and it may incur higher costs trying to maintain the current book of business or attracting new business. These additional costs are referred to as frictional costs. Because any source of risk can cause the company to incur these costs and they're generally not included in the DFA results, a company may want to incorporate some recognition of nonsystematic risk in its selection of risk and reward measures.

TABLE 10

Summary of Rankings

	CURRENT	A LITTLE STOCKS	SOME STOCKS	MORE STOCKS
CAPM	1	2	3	4
Excess Return Sharpe Ratio	4	3	2	1
Insurance Pricing Approach	4	3	1	2
Markowitz ($\tau = 0.075$)	4	3	2	1
Markowitz ($\tau = 0.04$)	4	3	1	2
Proportional Hazards	4	3	2	1
Proportional Hazards ($\rho = 1.5$) w/ CAPM	1	2	3	4
Proportional Hazards ($\rho = 2$) w/ CAPM	2	1	3	4

Assumption Regarding Distribution of ROEs

Most of the approaches based on the company's total risk (i.e., Excess Return Sharpe Ratio, Insurance Pricing, and Markowitz) assume that a single measure (standard deviation or variance) contains sufficient information regarding the distribution of ROEs to evaluate the riskiness of the strategy. The proportional hazards approach, on the other hand, makes no such assumption about the distribution of ROEs. In contrast, it uses the results of every iteration in calculating the adjusted mean. As such, in those situations in which the company has significant catastrophe or other asymmetric exposures, the proportional hazards approach would be preferred. In other situations, any of these methods could be used.

Risk Tolerance

In the Markowitz and proportional hazards methods, the company makes an explicit choice about its level of risk aversion. The insurance pricing approach could be modified to allow the user to select the level of risk aversion. By comparison, in the Excess Return Sharpe Ratio approach, the level of risk aversion is implicit in the calculation itself. In this method, the company is assumed to have the same level of risk aversion as the market in its entirety.

A company that believes its level of risk aversion is typical of the market might want to rely on the Excess Return Sharpe Ratio. On the other hand, a company that has a good understanding of its level of risk aversion might want to use one of the other methods.

One drawback of any of these methods that require a risk aversion parameter is the general lack of information available on the selection of the parameter. With the Markowitz approach, the level of risk aversion underlying its current strategy can be determined. For the proportional hazards approach, that parameter can't always be determined.

Concluding Observations

There are several ways to rank the strategies being evaluated in a dynamic financial analysis. Table 10 summarizes the relative rankings from each of these approaches.

Not surprisingly, the second through fifth results shown in Table 10 rank the strategies in roughly the same order. The first and second choices vary depending on the risk measure used (variance or standard deviation) and the level of risk tolerance, but there are no other differences in rank.

The CAPM results in the first row are quite different, since the higher rewards of the stock strategies don't offset the higher required return due to the increased market correlation. The combined approach with the lower risk aversion parameter provides the same rankings as the unadjusted CAPM. (Note that the unadjusted CAPM approach is the same as the combined approach with a risk aversion parameter of 1.00.) As risk aversion increases, greater preference is given to the stock strategies.

These differences in rankings and their consistency with the underlying assumptions of each method indicate the importance of understanding each approach, its assumptions, and their applicability to each company. ●